Reflexive Banach spaces not isomorphic to uniformly convex spaces.


Milman [C. R. (Doklady) Acad. Sci. URSS (N.S.) 20, 243–246 (1938)] and Pettis [Duke Math. J. 5, 249–253 (1939)] have shown independently that a Banach space which is isomorphic to a uniformly convex space must be reflexive; that is, for $\beta \in B^{**}$ there is a $b \in B$ such that $\beta(f) = f(b)$ for every $f \in B^*$. The natural question of whether the converse is true is answered in the negative in the present note; in addition the author shows that reflexivity is not sufficient even in the presence of separability and strict convexity of the space. The author deduces from his results that certain general ergodic theorems of Alaoglu and G. Birkhoff [Proc. Nat. Acad. Sci. U.S.A. 25, 628–630 (1939) and Ann. of Math. (2) 41, 293–309 (1940); cf. MR0000908 (1,148e)and 339] have a wider range of applicability than the one which was originally announced by those authors.

Reviewed by J. A. Clarkson

© Copyright American Mathematical Society 1941, 2006