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On the closed solution to some nonhomogeneous eigenvalue problems with $p$-Laplacian.
(English summary)


This paper is concerned with the nonlinear eigenvalue problems (E) $(|u'|^{p-2}u')' + \lambda|u|^{q-2}u = 0$ in $(0, T)$ with the Dirichlet, Neumann and periodic boundary value conditions. Here $T > 0$, $\lambda > 0$ and $p, q > 1$. First, the initial value problem for (E) is studied and the solution in closed form in terms of incomplete gamma functions is obtained. By using this, an explicit form of the whole spectrum for (E) is obtained under the indicated boundary conditions. In particular, for the Dirichlet condition, the set of eigenvalues of (E) is given by $\lambda_n(\alpha) = (2n\pi_{pq}/T)^q|\alpha|^{p-q}/(p^* q^* q^{-1})$ for any given $\alpha \neq 0$, where $\pi_{pq} = B(1/q, 1/p^*)$ and $p^* = p/(p - 1)$. It is also shown that the eigenvalues and eigenfunctions obtained in this paper and those provided by the Lyusternik-Shnirel'man theory coincide. The proofs are given by studying corresponding initial value problems and calculating incomplete beta functions.

Reviewed by Tetsutaro Shibata

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