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Approximation and entropy numbers of Volterra operators with application to Brownian motion. (English summary)


Entropy numbers and approximation numbers of integral operators between $L_p$- and $L_q$-spaces which are sharp for large classes of integral operators have been derived by many authors. However, they are often not asymptotically exact for Volterra integral operators. The authors give sharp estimates for the entropy and approximation numbers of weighted Volterra integral operators from $L_p$ to $L_q$:

$$T_{\rho,\psi}f(s) = \rho(s) \int_0^s \psi(t)f(t)\,dt.$$ 

Assuming certain integrability conditions, which are essentially shown to be optimal, the authors prove for the entropy numbers $e_n$,

$$e_n(T_{\rho,\psi}) \sim \|\rho\psi\|_r/n, \quad 1/r = 1 - 1/p + 1/q > 0.$$ 

The integrability assumptions are formulated in terms of $L_q$-norms of $\rho, \psi$ over dyadic intervals and taking suitable sums of them. For the approximation numbers, the main result (6.20) states (essentially):

$$a_n(T_{\rho,\psi}) \sim \|\rho\psi\|_r/n^\lambda,$$

where $\lambda - 1/r$ if $p \le q \le 2$ or $2 \le p \le q$, $\lambda = 1/2 + \min(1/q, 1/p')$ if $p \le 2 \le q$ and $\lambda = 1$ if $q < p$. By scale transformations, studied in Chapter 3, the estimates for $T_{\rho,\psi}$ are reduced to $T_{\rho} = T_{\rho,1}$, i.e. $\psi = 1$, and thus involve the ordinary integration operator. The upper and lower bounds for the entropy numbers are given in chapters 4 and 5, and the case of approximation numbers is considered in Chapter 6. The results generalize results of, e.g., Edmunds, Evans and Harris. A new part of the method of proof is the introduction of a probabilistic point of view by using decompositions of the Wiener process into a family of independent Brownian bridges and a piecewise linear process. Using such techniques, the authors consider in Chapter 7 general entropy based bounds for small ball probabilities.

Reviewed by Hermann König

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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