Mean dimension and widths of classes of functions on the line. (Russian)


Let $1 \leq p \leq \infty$, $r \in \mathbb{N}$. The author denotes by $W_p(\mathbb{R})$ the set of functions $x(\cdot) \in L_p(\mathbb{R})$ such that $x^{(r-1)}(\cdot)$ is a locally absolutely continuous function and $x^{(r)}(\cdot) \in L_p(\mathbb{R})$. $W_p(\mathbb{R})$ is a Banach space with the norm $\|x(\cdot)\|_{L_p(\mathbb{R})} + \|x^{(r)}(\cdot)\|_{L_p(\mathbb{R})}$. Let $W_p^r(\mathbb{R}) := \{x(\cdot) \in W_p(\mathbb{R}) : \|x^{(r)}(\cdot)\|_{L_p(\mathbb{R})} \leq 1\}$.

The author introduces the mean dimensions of Kolmogorov, Bernstein and linear types, and denotes them by $\overline{d}$, $\overline{b}$, $\overline{\lambda}$, respectively. The main result is as follows. Theorem 1: If $N > 0$, then

$$\overline{b}_N(W_p^r(\mathbb{R}), L_p(\mathbb{R})) = \overline{d}_N(W_p^r(\mathbb{R})),
\overline{L}_p(\mathbb{R}) = \overline{\lambda}_N(W_p^r(\mathbb{R}), L_p(\mathbb{R})) = A(p, r)N^{-r},$$

where $A(p, r) = 2^{-r}\|\hat{x}(\cdot)\|_{L_p([0,1])}$ and $x(\cdot)$ is the unique solution for the following extremal problem:

$$\|x(\cdot)\|_{L_p([0,1])} \to \sup \|x^{(r)}(\cdot)\|_{L_p([0,1])} \leq 1,$$

where $x^{(i)}((1 - (-1)^i)/2) = 0$, $0 \leq i \leq r - 1$. Reviewed by Ion Badea