On the absolutely continuous spectrum in a model of an irreversible quantum graph.


The authors study the absolutely continuous spectrum \( \sigma_{ac}(A_\alpha) \) of the family \( A_\alpha, \alpha > 0 \), of selfadjoint differential operators in the space \( L^2(\mathbb{R}^2) \), generated by the differential expression

\[
AU = -U''_{xx} + \frac{1}{2}(-U'_{qq} + q^2 U),
\]

and the transmission condition across the line \( x = 0 \)

\[
U'_x(+0, q) - U'_x(-0, q) = \alpha q U(0, q), \quad q \in \mathbb{R}.
\]

A certain Jacobi operator in the space \( l^2 \) is involved in the description of \( \sigma_{ac}(A_\alpha) \), namely

\[
\mathcal{J}_0(\mu): \{C_n\} \rightarrow \{d_{n+1}C_{n+1} + (2n + 1)\mu C_n + d_n C_{n-1}\},
\]

\[
d_n = n^{1/2}(n^2 - 1/4)^{1/4},
\]

where \( \mu > 0 \) is an auxiliary parameter. The main result of the paper states that

\[
\sigma_{ac}(A_\alpha) = \sigma_{ac}(A_0) \cup \sigma_{ac}(\mathcal{J}_0(\sqrt{2}/\alpha)),
\]

and

\[
m_{ac}(E; A_\alpha) = m_{ac}(E; A_0) + m_{ac}(E; \mathcal{J}_0(\sqrt{2}/\alpha)), \quad \text{a.e. } E \in \mathbb{R},
\]

where the symbol \( m_{ac} \) stands for the multiplicity function of the absolutely continuous spectrum. In this situation the operator \( A_0 \) admits separation of variables, which leads to the complete description of its spectrum: \( \sigma(A_0) = \sigma_{ac}(A_0) = [1/2, \infty) \), and

\[
m_{ac}(E; A_0) = 2n, \quad \text{for } E \in (n - 1/2, n + 1/2), \quad n \in \mathbb{N}.
\]

The possibility of extending the results to operators on quantum graphs is discussed.

Reviewed by Alexander M. Gomilko

References


Note: This list, extracted from the PDF form of the original paper, may contain data conversion errors, almost all limited to the mathematical expressions.

© Copyright American Mathematical Society 2006