This paper is concerned with Sturm-Liouville type problems for the equation
\[ L_p u + f(r,u) = 0 \] in \( I = [a,b] \) with boundary conditions
\[ \gamma_1 u^{(p-1)}(a) + \gamma_2 (r^\alpha u^{(p-1)})(a) = u_0, \]
\[ \gamma_3 u^{(p-1)}(b) + \gamma_4 (r^\alpha u^{(p-1)})(b) = u_1 \] (Bu = (u_0, u_1) for short), where
\[ L_p u = -r^{-\alpha} r^\alpha u^{(p-1)}', \]
\[ u^{(p-1)} \in \mathbb{R} \] (\( i = 1, \ldots, 4 \), \( j = 1, 2 \)) with \( \gamma_1^2 + \gamma_2^2 > 0, \gamma_3^2 + \gamma_4^2 > 0 \). The eigenvalue problem associated with (1)–(3) is
\[ L_p u + (q(r) + \lambda s(r)) u^{(p-1)} = 0 \] in \( I \) with the boundary condition
\[ Bu = (0,0) \.] Here \( q, s \in L^\infty(I) \) and ess inf \( s > 0 \). In Theorem 1, by using a concept of Prüfer-type transformation which is new for \( p > 1 \), it is shown that the problem (4),(5) has a countable number of simple eigenvalues \( \lambda_1 < \lambda_2 < \cdots, \lambda_n \to \infty \) as \( n \to \infty \), and there are no other eigenvalues. In addition, the corresponding eigenfunction \( u_n \) has \( n - 1 \) simple zeros in \( I^0 = (a,b) \). These results generalize a classical and well-known theorem for \( p = 2 \) to general \( p > 1 \). In Theorem 2, the global description of the Fučík spectrum \((\mu, \nu)\) of the problem
\[ L_p u + q(r) u^{(p-1)} + s(r) (\mu(u^+)^{(p-1)} - \nu(u^-)^{(p-1)}) = 0 \] in \( I \) with the boundary condition
\[ Bu = (0,0) \] is discussed. Based on these properties of the Fučík spectrum \((\mu, \nu)\), the existence of a solution to (1)–(3) is proved under appropriate conditions on \( f \) in Theorem 3. The proofs of Theorems 2 and 3 are also based on the Prüfer transformation.

Reviewed by Tetsutaro Shibata

References


*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*

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