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The limiting equation for Neumann Laplacians on shrinking domains. (English summary)


The author studies the problem of approximating the Neumann Laplacian on thin domains by differential operators on trees embedded in the domains. Let \( \{ \Omega_\varepsilon \}_{0 < \varepsilon \leq 1} \) be a family of connected open sets in \( \mathbb{R}^2 \) that shrinks to a tree \( \Gamma \) as \( \varepsilon \to 0 \). Let \( H_{\Omega_\varepsilon} \) be the Neumann Laplacian. For \( f \in H^1(\Omega_1) \), let \( f_\varepsilon \) be its restriction to \( \Omega_\varepsilon \). For \( z \in \mathbb{C} - [0, \infty) \), set \( u_\varepsilon = (H_{\Omega_\varepsilon} - z)^{-1} f_\varepsilon \). Under some assumptions on \( \Gamma \) and \( \Omega_\varepsilon \), the author shows that the limit function \( u_0 = \lim_{\varepsilon \to 0} u_\varepsilon \) satisfies the following differential equation on the edges of \( \Gamma \):

\[
-a(\sigma)^{-1} \frac{d}{d\sigma} \left( a(\sigma) \frac{du_0}{d\sigma} \right) - zu_0 = f_0(\sigma).
\]

Here \( \sigma \) is local arclength parametrisation of \( \Gamma \), and \( a(\sigma) \) is a function related to the thickness of \( \Omega_1 \). At the vertices of \( \Gamma \), \( u_0 \) satisfies Kirchhoff boundary conditions.

Reviewed by Julian Edward

References


*Note:* This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.

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