Weighted inequalities for positive operators. (English summary)


The purpose of this paper is to provide weighted inequalities of the form

\[
\left( \int_X (T f)^q u d\mu \right)^{1/q} \leq C \left( \int_Y f^p v d\nu \right)^{1/p},
\]

where \((T f)^q = \prod_{i=1}^n (T_i f^{r_i})^{q_i/r_i}\) for certain operators \(T_i\) defined on positive measurable functions, and \(q = q_1 + \cdots + q_n\). The main result follows the idea of Schur’s Lemma and it shows that if \(0 < g < \infty\) \(\nu\)-almost everywhere and \(0 < T g < \infty\) \(u\mu\)-almost everywhere, then

\[
\left( \int_X (T f)^q u d\mu \right)^{1/q} \leq C \left( \int_Y f^p v g d\nu \right)^{1/p},
\]

with

\[
v_g = \sum_{i=1}^n (q_i/q) g^{r_i-1} T_i^*(u(T g)^q/T_i(g^{r_i})).
\]

Moreover, under certain conditions, a converse result is also true. To prove this converse result, an extension to operators in Banach function spaces of the minimax principle introduced by G. Frobenius is presented and examples involving Hardy and Stieltjes operators are given in the last section.

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