On eigenvalue estimates for the weighted Laplacian on metric graphs. (English summary)

The main goal of this paper is to study the behavior of eigenvalues for the problem

$$-\lambda \Delta u = Vu, \; u(x_0) = 0, \; u'(v) = 0, \; v \in \partial G \setminus \{x_0\},$$

on a connected graph $G$ of finite total length, where $x_0 \in G$ is an arbitrary fixed vertex and $V = \nabla \in L^1(G)$. Let $\lambda_{n}^{\pm}$ denote the positive and negative eigenvalues of this problem. The author proves that

$$\lambda_{n}^{\pm} \leq n^{-2}|G| \int_{G} V_{\pm} dx \; \forall n \in \mathbb{N}, \; 2V_{\pm} := |V| \pm V,$$

which implies the Weyl-type asymptotics

$$n \sqrt{\lambda_{n}^{\pm}} \to ^{\pi^{-1}} \int_{G} \sqrt{V_{\pm}(x)} dx, \; n \to \infty.$$ 

Applications and generalizations of this result are also discussed. 

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