$$\sum_{n=1}^{\infty} \frac{1}{n^s} = \prod_{p} \left(1 - \frac{1}{p^s}\right)^{-1}$$

In mathematics, an Euler product is an infinite product expansion, indexed by prime numbers p, of a Dirichlet series. The name arose from the case of the Riemann zeta-function, where such a product representation was proved by Euler.

$$\sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{\pi^2}{6}$$

One of Euler's earliest triumphs was a solution to the so-called "Basel Problem" that had perplexed mathematicians for the better part of the previous century. Hundreds of years into its age the problem remained unsolved and anyone capable of summing the series was certain to make a major splash. When it happened in 1735, the splash was Euler's. The answer was not only a mathematical tour de force but a genuine surprise for the series sums to $\overline{\epsilon}$ a highly non-intuitive result.

$$\gamma = lim_{n\to\infty}\left(1 + \frac{1}{2} + \cdots + \frac{1}{n} - \log n\right)$$

The constant was first defined by Swiss mathematician Leonhard Euler in a paper De Progressionibus harmonicus observationes published in 1735. Euler used the notation C for the constant, and initially calculated its value to 6 decimal places. In 1781 he extended this calculation, publishing a value to 16 decimal places. In 1790 Italian mathematician Lorenzo Mascheroni introduced the notation γ for the constant, and attempted to extend Euler's calculation still further, to 32 decimal places, although subsequent calculations showed that he had made errors in the 20th-22nd, 31st and 32nd decimal places.

$$e = \sum_{n=1}^{\infty} \frac{1}{n!}$$

The first known use of the constant, represented by the letter b, was in correspondence from Gottfried Leibniz to Christiaan Huygens in 1690 and 1691. Leonhard Euler started to use the letter e for the constant in 1727, and the first use of e in a publication was Euler's Mechanica (1736). While in the subsequent years some researchers used the letter c, e was more common and eventually became the standard.

The exact reasons for the use of the letter e are unknown, but it may be because it is the first letter of the word exponential. Another possibility is that Euler used it because it was the first vowel after a, which he was already using for another number, but his reason for using vowels is unknown. It is unlikely that Euler chose the letter because it is his last initial, since he was a very modest man, and tried to give proper credit to the work of others.

$$e^{i\theta} = \cos\theta + i\sin\theta$$

Euler's formula, named after Leonhard Euler, is a mathematical formula in complex analysis that shows a deep relationship between the trigonometric functions and the complex exponential function.

Euler's formula was proven for the first time by Roger Cotes in 1714 in the form ln(cos(x) + i sin(x)) = ix (where "ln" means natural logarithm, i.e. log to base e). It was Euler who published the equation in its current form in 1748, basing his proof on the infinite series of both sides being equal. Neither of these men saw the geometrical interpretation of the formula: the view of complex numbers as points in the complex plane arose only some 50 years later.

$$e^{i\pi} + 1 = 0$$

Gauss is reported to have commented that if this formula was not immediately apparent to a student on being told it, the student would never be a first-class mathematician.

After proving the identity in a lecture, Benjamin Peirce, a noted nineteenth century mathematician and Harvard professor, said, "It is absolutely paradoxical; we cannot understand it, and we don't know what it means, but we have proved it, and therefore we know it must be the truth."

Stanford mathematics professor Keith Devlin says, "Like a Shakespearean sonnet that captures the very essence of love, or a painting that brings out the beauty of the human form that is far more than just skin deep, Euler's equation reaches down into the very depths of existence."