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LEONHARD EULER • 1707 • 1783 •
DEUTSCHE DEMOKRATISCHE REPUBLIK

Euler’s Disk

Euler’s disk, named after Leonhard Euler, is a circular disk that spins, without slipping, on a surface. The canonical example is a coin spinning on a table. It is universally observed that a spinning Euler’s disk ultimately comes to rest; and it does so quite abruptly, the final stage of motion being accompanied by a whirring sound of rapidly increasing frequency. As the disk rolls, the point P of rolling contact describes a circle that oscillates with a constant angular velocity ω. If the motion is non-dissipative, ω is constant and the motion persists forever, contrary to observation.

Euler’s Rule for Finding Amicable Numbers

Euler’s rule, named after Leonhard Euler, is a generalization of Thâbit ibn Kurrah rule for finding amicable numbers. If $a = 2m\times(2n-m + 1) - 1$, $b = 2n\times(2n-m + 1) - 1$, and $c = 2n+m\times(2n-m + 1)2 - 1$ are all prime, for integers $0 < m < n$, then $2n \times a \times b$ and $2n \times c$ are amicable. This hypothesis is satisfied for the pairs (m,n) = (1,2), (3,4), (6,7), (1,8), and (29,40), but for no other pairs with $n < 2500$. The first three of these pairs yield the three pairs of amicable numbers discovered using the Thâbit ibn Kurrah rule.

Euler Brick

In mathematics, an Euler brick, named after the famous mathematician Leonhard Euler, is a cuboid with integer edges and also integer face diagonals. A primitive Euler brick is an Euler brick with its edges relatively prime.

Alternatively stated, an Euler Brick is a solution to the following system of diophantine equations:

$$a^2 + b^2 = d^2$$
$$b^2 + c^2 = e^2$$
$$a^2 + c^2 = f^2$$

The smallest Euler brick has edges

(a,b,c) = (240,117,44)

and face polyhedron diagonals

267, 244, and 125.

Paul Halcke discovered it in 1719.

Other solutions are: Given as: length (a, b, c)

(275, 252, 240), (693, 480, 140), (720, 132, 85), (792, 231, 160)

Euler found at least two parametric solutions to the problem, but neither give all solutions.

Given an Euler brick with edges (a,b,c), the triple (bc,ac,ab) constitutes an Euler brick as well.

Euler Integrals

In mathematics, there are two types of Euler integral:

1. Euler integral of the first kind: the Beta function

$$B(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}\,dt = \frac{\Gamma(x)\Gamma(y)}{\Gamma(x+y)}$$

2. Euler integral of the second kind: the Gamma function

$$\Gamma(z) = \int_0^\infty t^{z-1}\,e^{-t}\,dt$$

For positive integers m and n

$$B(n,m) = \frac{(n-1)!(m-1)!}{(n+m-1)!} = \frac{n+m}{nm\binom{n+m}{n}}$$

$$\Gamma(n) = (n-1)!$$

Euler Maclaurin Formula's

The Euler-Maclaurin formula provides a powerful connection between integrals and sums. It can be used to approximate integrals by finite sums, or conversely to evaluate finite sums and infinite series using integrals and the machinery of calculus. The formula was discovered independently by Leonhard Euler and Colin Maclaurin around 1735. Euler needed it to compute slowly converging infinite series while Maclaurin used it to calculate integrals.

If n is a natural number and f(x) is a smooth (meaning: sufficiently often differentiable) function defined for all real numbers x between 0 and n, then the integral

$$I = \int_0^n f(x)\,dx$$

can be approximated by the sum

$$S = \frac{f(0)}{2} + f(1) + \cdots + f(n-1) + \frac{f(n)}{2} = \frac{f(0) + f(n)}{2} + \sum_{k=1}^{n-1} f(k)$$

The Euler-Maclaurin formula provides expressions for the difference between the sum and the integral in terms of the higher derivatives f(k) at the end points of the interval 0 and n. For any natural number p, we have

$$S - I = \sum_{k=1}^{p} \frac{B_{2k}}{(2k)!} \left(f^{(2k-1)}(n) - f^{(2k-1)}(0) \right) + R$$

where B1 = −1/2, B2 = 1/6, B3 = 0, B4 = −1/30, B5 = 0, B6 = 1/42, B7 = 0, B8 = −1/30, ... are the Bernoulli numbers, and R is an error term which is normally small for suitable values of p.

Euler Integration

In mathematics and computational science, Euler integration (or the Euler method) is a numerical procedure for solving ordinary differential equations (ODEs) with a given initial value. It is the most basic kind of explicit numerical integration for ordinary differential equations.

Derivation:

We want to approximate the solution of the initial value problem

$$y'(t) = f(t, y(t)), \qquad y(t_0) = y_0,$$

by using the first two terms of the Taylor expansion of y. One step of Euler Integration from tn to tn+1 = tn + h is

$$y_{n+1} = y_n + hf(t_n, y_n).$$

The Euler method of integration is explicit, i.e. the solution yn + 1 is an explicit function of yi for i<n+1.

While Euler integration integrates a first order ODE, any ODE of order N can be represented as a first-order ODE in more than one variable by introducing N – 1 further variables, y', y'', ..., y(N), and formulating N first order equations in these new variables. The Euler method can be applied to the vector

$$\mathbf{y}(t) = (y(t), y'(t), y''(t), ..., y^{(N)}(t))$$

to integrate the higher-order system.


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e-k+f=2
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