Homework 3 **Math 5590H**

Due by Tuesday, September 12

1.3.14. Let p be a prime. Show that a permutation $\sigma \in S_n$ has order p iff σ is a product of disjoint p-cycles. Show by example that this doesn't need to be so if p is not prime.

3.5.4,5. (a) For any $n \in \mathbb{N}$, prove that S_n is generated by any n-cycle $\rho = (i_1, i_2, \ldots, i_n)$ and the transposition $\tau = (i_1, i_2)$.

(b) If p is prime, show that $S_p = \langle \tau, \rho \rangle$ where ρ is any p-cycle and τ is any transposition.

2.4.7. Prove that the subgroup of S_4 generated by $\tau = (1,2)$ and $\rho = (1,3)(2,4)$ is isomorphic to D_8.

2.2.7. For all $n \geq 3$, prove that $Z(D_{2n}) = \{1, r^{n/2}\}$ if n is even and $Z(D_{2n}) = 1$ if n is odd.

A1. Prove that for any $n \in \mathbb{N}$ and field F, $Z(\mathrm{GL}_n(F))$ is the set of scalar matrices cI, $c \in F^*$.

3.2.8. Prove that if H and K are finite subgroups of G whose orders are relatively prime, then $H \cap K = 1$.

3.2.22. Determine the last two digits of $3^{3^{100}}$. (*Hint:* $\varphi(100) = (4-2)(25-5) = 40$, where φ is Euler's totient function.)