10% **1.** If two subgroups $H, K \leq G$ have coprime orders (i.e. $\gcd(|H|, |K|) = 1$) prove that $|HK| = |H| \cdot |K|$.

20% **2.** Let $H \leq G$, $|G : H| = n$, and assume that H contains no nontrivial (i.e. $\neq 1$) subgroups that are normal in G. Prove that G is isomorphic to a subgroup of S_n. (*Hint:* Consider the action of G on the set G/H of left cosets of H.)

20% **3.** Let H be a subgroup of finite index in a group G. Prove that the number of subgroups conjugate to H divides $|G : H|$. (*Hint:* Consider the action of G on the set of conjugates of H.)

15% **4.** Determine whether the groups $\mathbb{Z}_{36} \times \mathbb{Z}_{12} \times \mathbb{Z}_{10}$ and $\mathbb{Z}_{60} \times \mathbb{Z}_{18} \times \mathbb{Z}_4$ are isomorphic.

5. Let $G = \langle a, b, c \mid a^5 = b^5 = c^4 = 1,\ ab = ba,\ ca = bc,\ cb = a^4c \rangle$.

15% (a) Represent G as a semidirect product of abelian groups and find the factors of a (any) composition series of G.

10% (b) Represent G is a quotient group F/N of a free group F.

20% **6.** If P is a Sylow p-subgroup of a finite group G and $N_G(P)$ is normal in G, prove that G has a unique Sylow p-subgroup. (*Hint:* Notice that P is a Sylow subgroup of $N_G(P)$.)

20% **7.** Find all, up to isomorphism, groups of order $155 = 31 \cdot 5$.

20% **8.** Prove that all groups of order $1452 = 2^2 \cdot 3 \cdot 11^2$ are solvable. (*Hint:* Pay attention to n_{11}, the number of Sylow 11-subgroups.)