

Groups

1. Definition and elementary properties of groups.
2. The groups \mathbb{Z} , \mathbb{Z}_n , \mathbb{Z}_n^* , V_4 , D_{2n} , Q_8 , the free group F_X , the symmetric group S_X .
3. Subgroups. Subgroups, generated by sets.
4. Cyclic groups. Subgroups of \mathbb{Z} and of \mathbb{Z}_n .
5. The group S_n . Transpositions. Cyclic decompositions of permutations. Generators of S_n . Even and odd permutations. The group A_n .
6. Cosets and index of a subgroup. Lagrange's theorem. Counting principles.
7. Normal subgroups. Quotient groups.
8. Normalizers and centralizers. The center of a group.
9. Simple groups, composition series, the Jordan-Hölder theorem.
10. Conjugation, conjugacy classes.
11. Conjugacy classes in S_n . Simplicity of A_n for $n \geq 5$.
12. Homomorphisms of groups. Kernel. Isomorphisms.
13. Isomorphism theorems.
14. Groups defined by generators and relations as factors of free groups.
15. Reduction of a homomorphism to a quotient group.
16. Actions of groups on sets. Orbits and stabilizers.
17. The left regular action of a group on itself. Cayley's theorem.
18. The action of a group on itself by conjugations.
19. Direct products of groups, external and internal.
20. The Chinese remainder theorem.
21. The fundamental theorem of finite abelian groups.
22. The group \mathbb{Z}_n^* .
23. Groups of automorphisms of groups.
24. The groups $\text{Aut}(\mathbb{Z}_n)$, $\text{Aut}(\mathbb{Z}_p^n)$.
25. Inner and outer automorphisms of groups.
26. Characteristic subgroups.
27. Semidirect products of groups, internal and external.
28. p -groups.
29. Sylow's theorems.
30. Groups of orders pq , pqr , p^2q , pq^k , p^2q^2 .
31. Methods of proving that a finite group is non-simple.
32. Commutators, derived subgroups, and solvable groups.
33. Central series, nilpotent groups.