Math 4181H **Homework 6**

Due by Tuesday, October 14

Chapter 8, p. 140:

6. Let $B \subseteq \mathbb{R}$ and let A be a dense subset of B.

5pt (b) Prove that if f and g are continuous on B and $f(x) = g(x)$ for all $x \in A$, then $f = g$ on B (that is, $f(x) = g(x)$ for all $x \in B$).

5pt (c) Prove that if f and g are continuous on B and $f(x) \geq g(x)$ for all $x \in A$, then $f \geq g$ on B (that is, $f(x) \geq g(x)$ for all $x \in B$). Can "$\geq$" be replaced by "$>$"?

Chapter 6, pp. 120-121:

5pt **4.** Give an example of a function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ that is continuous nowhere, but $|f|$ is continuous everywhere.

5pt **5.** For each point $a \in \mathbb{R}$ find a function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ which is continuous at a but is discontinuous at all other points of $\mathbb{R}$.

5pt **6.** (b) Find a function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ that is discontinuous at $0, 1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ but continuous at all other points.

5pt (a) Find a function $f\colon \mathbb{R} \longrightarrow \mathbb{R}$ that is discontinuous at $1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \ldots$ but continuous at all other points.

Chapter 7, pp. 130-132:

5pt **10.** Suppose f and g are continuous on $[a, b]$, $f(a) < g(a)$, and $f(b) > g(b)$. Prove that $f(x) = g(x)$ for some $x \in [a, b]$.

10pt **8.** Suppose that f and g are continuous on an interval I, that $f^2 = g^2$, and that $f(x) \neq 0$ for all x. Prove that either $f = g$ or $f = -g$. (*Hint:* Consider the function $h = g/f$.)

10pt **16.** (a) Suppose that f is continuous on (a, b) and $\lim_{x\to a^+} f(x) = \lim_{x\to b^-} f(x) = +\infty$. Prove that f has a minimum on all of (a, b). (*Hint:* Choose a **closed** subinterval of (a, b) such that f is "large" outside of it.)

5pt **18.** Suppose f is a continuous function on $\mathbb{R}$ with $f(x) > 0$ for all x, and $\lim_{x\to-\infty} f(x) = \lim_{x\to+\infty} f(x) = 0$. Prove that f attains its maximal value in $\mathbb{R}$.

Chapter 8, Appendix, p. 146:

5pt **2.** (a) If f and g are uniformly continuous on A, then so is $f + g$.

5pt (b) If f and g are uniformly continuous and bounded on A, then fg is unformly continuous on A.

5pt (c) Show that the conclusion of (b) fails if one of f, g is unbounded.

5pt (d) Suppose that f is uniformly continuous on A, g is uniformly continuous on B, and $f(A) \subseteq B$. Prove that $g \circ f$ is uniformly continuous on A.