Math 4181H **Midterm 2**

In your solutions you may use any results proven in class, in homework, or in the lecture notes.

20% **1.** Find $\lim \sqrt[n]{100n^{100} + 2^n}$ (and justify your answer, of course).

20% **2.** Let (x_n) be a sequence satisfying $|x_n - x_m| < 1/\min\{n, m\}$ for all n, m. Prove that this sequence converges.

20% **3.** If $\lim_{x\to 0^+} f(x) = \infty$, prove that $\lim_{x\to+\infty} \frac{1}{f(1/x)} = 0$. (Proofs like $\frac{1}{0^+} = +\infty$ are not accepted.)

20% **4.** Let $f\colon A \longrightarrow \mathbb{R}$ be a monotone function and let $a \in A$ be a limit point of both $A \cap (-\infty, a)$ and $A \cap (a, +\infty)$. Suppose there are sequences (x_n) and (y_n) in $A \setminus \{a\}$ such that (x_n) is increasing to a, (y_n) is decreasing to a, and $\lim f(x_n) = \lim f(y_n)$. Prove that f is continuous at a.

5. Let $f\colon [0, +\infty) \longrightarrow \mathbb{R}$ be a continuous function with the property that a finite $\lim_{x\to+\infty} f(x)$ exists.

20% (a) Prove that f is bounded. (*Hint:* Choose M large enough and consider the intervals $[0, M]$ and $[M, +\infty)$ separately.)

5% (b) Does f have to attain its maximal value?