

Math 4181H**Midterm 3**

In your solutions you may use any results proven in class, in homework, or in the lecture notes.

1. Let f be a function differentiable in a neighborhood of a point a with $f'(a) > 0$.
- 10% (a) Prove that f is strictly increasing at a (that is, for all x in a neighborhood of a , $f(x) < f(a)$ if $x < a$ and $f(x) > f(a)$ if $x > a$).
- 20% (b) If f' is continuous at a , prove that f is strictly increasing in a neighborhood of a .
- 20% 2. Prove that for any $n \in \mathbb{N}$ and any $x_1, \dots, x_n > 0$, $\log\left(\frac{x_1 + \dots + x_n}{n}\right) \geq \frac{\log x_1 + \dots + \log x_n}{n}$.
- 20% 3. Suppose f is differentiable on an interval I and $f'(x) \neq 0$ for all $x \in I$; prove that f is strictly monotone on I .
- 15% 4. (a) Prove that for any $x > 0$, $\sin x < x$.
- 15% (b) Prove that for every $\varepsilon > 0$ there exists $\delta > 0$ such that $\sin x > (1 - \varepsilon)x$ for all $x \in (0, \delta)$.
- 20% 5. Find $(f^{-1})''(f(a))$ in terms of the derivatives of f at a .