Math 4181H **Midterm 4**

In your solutions you may use any results proven in class, in homework, in the textbook, or in the lecture notes.

20% **1.** If f is an integrable function on $[a,b]$ with the property that the set $\{x : f(x) = 0\}$ is dense in $[a,b]$, prove that $\int_a^b f = 0$.

20% **2.** Derive the mean value theorem for integrals (if f is continuous on $[a,b]$ then $\int_a^b f = f(c)(b-a)$ for some $c \in [a,b]$) from the (Lagrange's) mean value theorem for derivatives.

20% **3.** Find a primitive (antiderivative) function F on $\mathbb{R}$ of the function $f(x) = \begin{cases} x, & x \leq 0 \\ \sin x, & x \geq 0 \end{cases}$. (Notice that F must be differentiable.)

20% **4.** Prove that the improper integral $\int_0^{+\infty} e^{-x^2} \sin x \, dx$ converges.

20% **5.** Find $\displaystyle\lim_{x\to 0} \frac{\sin(x^2) - x^2}{(\cos x - 1)^3}$.

20% **6.** Assuming that the function $f(x) = (\sin x)/x$ for $x \neq 0$, $f(0) = 1$, is 100 times differentiable at 0 (don't prove this), find $f^{(100)}(0)$.