A graph is a structure that defines pairwise relationships within a set to objects. The objects are the vertices, and the pairwise relationships are the edges: $X$ is related to $Y$ if and only if $XY$ is an edge.

A graph is a picture consisting of:

- **Vertices**: dots
- **Edges**: lines
  - The edges do not have to be straight lines. But they have to connect two vertices.
- **Loop**: an edge connecting a vertex back with itself
Nodes = Companies
Edge = portfolios own each others stock

Mathematicians have found to produce random graphs that have similar structures to many real-world graphs. These allow us to analyze the structure to find ways to solve real-world problems.
The following graph represents roads on a city grid. How many different routes are there from A to B (Only going North or East)?

What do mathematicians look for in graphs?

How can you get from one place to another?
How hard is it to send a message from one node to another? to all the others?
How does information (a rumour, or a disease, or a fad) spread across a network?
Are some nodes "more important" than others?
Can you draw the graph without "crossing edges"?

Draw the cables/pipes between three houses (on right) and gas, electric and water companies on the left. Try not to let the pipes cross.

Do the same for these four houses and two utilities.

Real world application: Design computer chips. Need to connect many input sources with many output sources around the edge of a chip. Each crossing adds cost as well as heat to the chip. You would like as few crossings as possible. How to design it?

Notation for graphs

This graph has six vertices A, B, C, D, E, and F and eight edges. The edges can be described by giving the two vertices that are connected by the edge.

List all the edges: (as a pair of vertices)
Definitions and Terminology

Two vertices are said to be adjacent if

The degree of a vertex is

A path is a sequence of vertices with the property that each vertex in the sequence is adjacent to the next one. The key requirement in a path is that an edge can be part of a path only once.

A circuit is....
Attachments

- Web Pages as Graphs
- Euler Circuit
- TheHousesAndUtilitiesCrossingProblem.nbp