Definitions and Terminology

Two vertices are said to be **adjacent** if

The degree of a vertex is

\[
\text{deg}(A) = 2 \\
\text{deg}(C) = 3
\]

A path is a sequence of vertices with the property that each vertex in the sequence is adjacent to the next one. The key requirement in a path is that an edge can be part of a path only once.

A **circuit** is...

A cycle is a circuit with no repeated vertex.

A graph is **connected**, if

There is a path between any two vertices.

A graph that is not connected is said to be **disconnected**.

A disconnected graph is made up of separate **components**.
Sometimes in a connected graph there is an edge such that if we were to erase it, the graph would become disconnected—such an edge is called a **bridge**.

What edges are the bridges in this graph?

- The edge \( (F, B) \) is a bridge.
- The edge \( (F, G) \) is also a bridge.

5-Room Puzzle: Is there a way to visit every room in this house without using the same door twice?

Different Map

Works Why?

Impossible
How is this a graph?

vertices = rooms
edges = if there is a door between two rooms

The question becomes... Can you visit every vertex without...

Can you visit every edge exactly once and return to start?

Is there a circuit that goes on every edge exactly one time?

**Euler Circuit:** A circuit in a graph that crosses every edge exactly once and ends up where it started.

**Euler Path:** A path that crosses every edge exactly once (doesn't end where it started).

Try to trace out all of the edges of this graph without repeating any edges or lifting your pencil to start someplace new.
A Unicursal Drawing of a graph is one which draws each edge of the graph without going over the same edge twice.

Which of these graphs have one?

Have to start on odd, have to end on odd. Every vertex in middle must have even degree.
4

"Doesn't Work."

12

Can I draw this in one path?

Starting vertex uses odd # of edges
Ending vertex uses odd # of edges
Every other vertex must have even # of edges.

Impossible more than 2 "odd" vertices.
An Euler path is a path that passes through every edge of a graph once and only once. The graph shown in (a) does not have an Euler path; the graph in (b) has several Euler paths. One of them is L, A, R, D, A, R, D, L, A.
Euler’s **Circuit** Theorem
• If a graph is *connected*, and every vertex is *even*, then it has an Euler circuit (at least one, usually more).
• If a graph has *any* odd vertices, then it does not have an Euler circuit.

Start and end on same vertex.

Euler’s **Path** Theorem
• If a graph is *connected*, and has exactly *two* odd vertices, then it has an Euler path (at least one, usually more). Any such path must start at one of the odd vertices and end at the other one.
• If a graph has *more than two* odd vertices, then it cannot have an Euler path.

Start and end on different vertices.
Attachments

- Web Pages as Graphs
- Euler Circuit
- TheHousesAndUtilitiesCrossingProblem.nbp