Chapter 7 Spanning Trees Homework Problems  Due Monday April 1

2. For each graph, determine whether the graph is a tree or not. If the graph is not a tree, give the reason(s) why not.
(a) The graph shown in Fig. 7.32(a)
(b) The graph shown in Fig. 7.32(b)
(c) The graph shown in Fig. 7.32(c).

In Exercises 5 through 8, assume that $G$ is a graph with no loops or multiple edges, and choose the option that best applies:
(I) $G$ is definitely a tree (explain why); (II) $G$ is definitely not a tree (explain why); or (III) $G$ may or may not be a tree (in this case, give two examples of graphs that fit the description—one a tree and the other one not).

5. (a) $G$ has 8 vertices and 10 edges.

(b) $G$ has 8 vertices and 7 edges.

c) $G$ has 8 vertices and is connected, and every edge in $G$ is a bridge.
(d) $G$ has 8 vertices, and there is exactly one path from any vertex to any other vertex.

(e) $G$ has 8 vertices and 7 bridges.
7. (a) $G$ has 8 vertices and no circuits.

(b) $G$ has 8 vertices, 7 edges, and exactly one circuit.

(c) $G$ has 8 vertices, 7 edges, and no circuits.

11. Consider the network shown in Fig. 7-33.

(a) Find a spanning tree of the network.

(b) Calculate the redundancy of the network.

\[ \text{FIGURE 7-33} \]
12. Consider the network shown in Fig. 7-34.
   (a) Find a spanning tree of the network.
   (b) Calculate the redundancy of the network.

17. (a) How many different spanning trees does the network shown in Fig. 7-39(a) have?
   (b) How many different spanning trees does the network shown in Fig. 7-39(b) have?

Do not list them all, but rather find a way to count them.
19. For the network shown in Fig. 7-41,
   (a) find the MST of the network using Kruskal's algorithm.
   (b) give the weight of the MST found in (a).

   ![Figure 7-41](image)

22. For the network shown in Fig. 7-44,
   (a) find the MST of the network using Kruskal's algorithm.
   (b) give the weight of the MST found in (a).

   ![Figure 7-44](image)
25. The 3 by 4 grid shown in Fig. 7-47 represents a network of streets (3 blocks by 4 blocks) in a small subdivision. For landscaping purposes, it is necessary to get water to each of the corners by laying down a system of pipes along the streets. The cost of laying down the pipes is $40,000 per mile, and each block of the grid is exactly half a mile long. Find the cost of the cheapest network of pipes connecting all the corners of the subdivision. Explain your answer. *(Hint: First determine the number of blocks in the MST.)*

![Figure 7-47](image)

41. Give an example of a graph with $N = 11$ vertices and $M = 10$ edges having

(a) exactly one circuit.
(b) exactly two circuits.
(c) exactly three circuits.
51. (a) How many spanning trees does the network shown in Fig. 7-65(a) have?
(b) How many different spanning trees does the network shown in Fig. 7-65(b) have?
(c) How many different spanning trees does the network shown in Fig. 7-65(c) have?