# Chapter 1
The Mathematics of Voting
The Paradoxes of Democracy

- Elections involving ballots ranking several candidates.
- Consider many examples of balloting in society.
- Understand several different methods for determining a winner.
- Understand several different fairness criteria.
- Arrows' impossibility theorem.

HW:

(66) Pizza Problem

How many different Pizzas are there with 6 Topping choices.
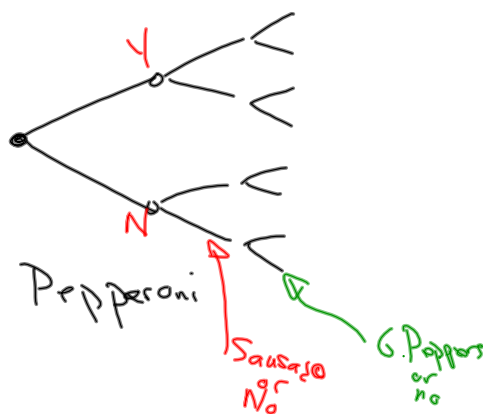
formal mathematical theory of probability.]

78. A factory assembles car stereos. From random testing at the factory, it is known that, on the average, 1 out of every 50 car stereos will be defective (which means that the probability that a car stereo randomly chosen from the assembly line will be defective is 0.02). After manufacture, car stereos are packaged in boxes of 12 for delivery to the stores.

(a) What is the probability that in a box of 12 car stereos, there are no defective ones? What assumptions are you making?

(b) What is the probability that in a box of 12 car stereos, there is at most 1 defective one?

$$Pr(Bad) = .02$$
$$Pr(Good) = .98 \quad \text{for each Box.}$$

$$Pr\left(\text{all } 12 \text{ are}\atop good\right) = \underset{1^{st}}{.98} \, \underset{2^{nd}}{.98} \, \underset{3^{rd}}{.98} \, \cdots \, \underset{12^{th}}{.98}$$

$$Pr\left(\text{exactly}\atop 1 \text{ is Bad}\right) = \underline{.98} \, \underline{.98} \, \underline{.98} \, \underline{.02} \, \underline{.98} \, \underline{.98} \cdots$$

$$(.98)^{11}(.02)^{1} \leftarrow \text{prob } 4^{th} \text{ radio Bad}$$

$$= 12 \cdot (.98)^{11}(.02)^{1}$$

12 possible positions for Bad radio

Pick a class President
from   Alice
        Bill
        Charlie
        Doug

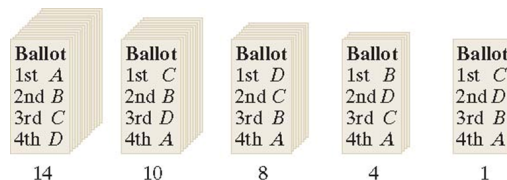Most 1st place votes is winner.

or

Method of Elimination (Vote them off the island)

- **Preference ballots**

    A ballot in which the voters are asked to rank the candidates in order of preference.
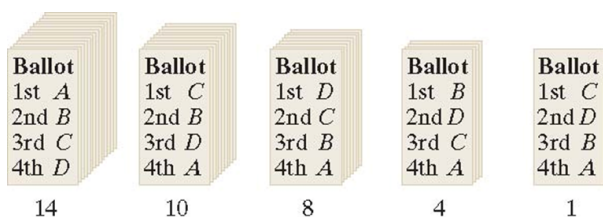- **Linear ballot**

    A ballot in which ties are not allowed.

Ballot	Ballot	Ballot	Ballot	Ballot
1st A	1st C	1st D	1st B	1st C
2nd B	2nd B	2nd C	2nd D	2nd D
3rd C	3rd D	3rd B	3rd C	3rd B
4th D	4th A	4th A	4th A	4th A
14	10	8	4	1

14 people prefered this order

37 total votes cast

Ballot	Ballot	Ballot	Ballot	Ballot
1st A	1st C	1st D	1st B	1st C
2nd B	2nd B	2nd C	2nd D	2nd D
3rd C	3rd D	3rd B	3rd C	3rd B
4th D	4th A	4th A	4th A	4th A
14	10	8	4	1

Who wins the election?   Most 1st place votes wins

A – 14
B – 4   C – 11
              D – 8   A wins.

Are most people happy about the result?

Suppose B + D dropped out

Between A + C who wins?

A – 14 votes   C – 23 votes.

More than 1/2 people prefer C over A.

But A won.

Assumptions about Ballots

- The first is that a voter's preference are **transitive**, i.e., that a voter who prefers candidate A over candidate B and prefers candidate B over candidate C automatically prefers candidate A over C (if B were not running).
- Secondly, that the relative preferences of a voter are not affected by the elimination of one or more of the candidates.

Methods of Choosing a Winner
based on all the preference ballots

- **Plurality method**
    Election of 1st place votes
- **Plurality candidate**
    The Candidate with the most 1st place votes

Plurality means more than any other candidate
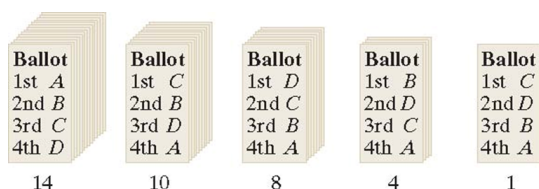Majority means more than half the voter
(more than all others combined)

- **Majority rule** or Criterion
    The candidate with a more than half the votes should be the winner.
- **Majority candidate**
    The candidate with the majority of 1st place votes .

Criterion is an outcome that should happen
but doesn't always.
depending on the method.

Criterion for selecting a Winner
(Things that "should" be true about an election)

The Majority Criterion

If candidate X has a *majority* of the 1st place votes, then candidate X should be the *winner* of the election.
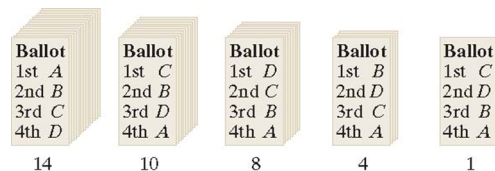
not just plurality

Ballot	Ballot	Ballot	Ballot	Ballot
1st A	1st C	1st D	1st B	1st C
2nd B	2nd B	2nd C	2nd D	2nd D
3rd C	3rd D	3rd B	3rd C	3rd B
4th D	4th A	4th A	4th A	4th A
14	10	8	4	1

Criterion for selecting a Winner
(Things that "should" be true about an election)

The Condorcet Criterion

If candidate X is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate X should be the winner of the election.

Ballot	Ballot	Ballot	Ballot	Ballot
1st A	1st C	1st D	1st B	1st C
2nd B	2nd B	2nd C	2nd D	2nd D
3rd C	3rd D	3rd B	3rd C	3rd B
4th D	4th A	4th A	4th A	4th A
14	10	8	4	1

Look at C

C vs A   C vs B   C vs D

23 - 14   10   14   14   8
           8   4   10
           1
           19 - 18   25 - 12

Condorcet Candidate.
A candidate who would win in a head to head vote against every other candidate

http://en.wikipedia.org/wiki/Marquis_de_Condorcet

Attachments

- Heisman Trophy Winner Selection

- Alternate Voting Methods for Presidential Primaries

- Results of Bush, Gore, Nader Presidential Vote in 2000

- Wikipedia Article on Voting Methods and Criteria

- Monotonicity Criterion

- Wikipedia Voting Systems Page

- wikipedia Arrows Impossibility Theorem

- Wikipedia Page on Kenneth Aarow

- Nice Web Page to Compare Several Types of Voting Methods