Chapter 1
The Mathematics of Voting
The Paradoxes of Democracy

- Elections involving ballots ranking several candidates.
- Consider many examples of balloting in society.
- Understand several different methods for determining a winner.
- Understand several different fairness criteria.
- Arrows’ impossibility theorem.

HW:
6b. Pizza Problem
How many different Pizzas are there with 6 Topping choices.
A factory assembles car stereo systems randomly. Of every 100 car stereo systems, 2 are defective (which means that the probability that a car stereo randomly selected is a defective one is 0.02). After manufacturers, six stereo systems are packaged in boxes of 12 for delivery to the stores.

What is the probability that in a box of 12 car stereos, there are no defective ones? What assumption must you make in order to answer this question?

What is the probability that in a box of 12 car stereos, there is at most 1 defective one?

\[
\begin{align*}
\Pr(\text{Bad}) &= 0.02 \\
\Pr(\text{Good}) &= 0.98
\end{align*}
\]

\[
\Pr(\text{all Good}) = \frac{0.98}{1} \cdot \frac{0.98}{2} \cdot \frac{0.98}{3} \cdots \frac{0.98}{12}
\]

\[
\Pr(\text{exactly 1 in Bad}) = \frac{0.98}{1} \cdot \frac{0.98}{2} \cdot \frac{0.02}{3} \cdots \frac{0.98}{12}
\]

\[
= \frac{12 \cdot (0.98)^{11} \cdot (0.02)}{12!}
\]

possible positions for Bad radio

Pick a class President
from Alice
Bill
Charlie
Doug

Most 1st place votes is winner.

or Method of Elimination (Vote them off the island)
• **Preference ballots**
  A ballot in which the voters are asked to rank the candidates in order of preference.

• **Linear ballot**
  A ballot in which ties are not allowed.

Who wins the election?

Most 1st place votes wins

\[
\begin{align*}
\text{A} & : \ 14 \\
\text{B} & : \ 8 \\
\text{C} & : \ 4 \\
\text{D} & : \ 1
\end{align*}
\]

A wins.

Are most people happy about the result?

Suppose B and D dropped out.

Between A and C, who wins?

\[
\begin{align*}
\text{A} & : \ 14 \text{ votes} \\
\text{C} & : \ 23 \text{ votes} \\
\end{align*}
\]

More than 1/2 people prefer C over A.

But A won.
Assumptions about Ballots

• The first is that a voter’s preference are transitive, i.e., that a voter who prefers candidate A over candidate B and prefers candidate B over candidate C automatically prefers candidate A over C (if B were not running).
• Secondly, that the relative preferences of a voter are not affected by the elimination of one or more of the candidates.

Methods of Choosing a Winner based on all the preference ballots

• Plurality method
  - Election of 1st place votes
• Plurality candidate
  - The Candidate with the most 1st place votes

Plurality means more than any other candidate
Majority means more than half the voters (more than all others combined)

• Majority rule
  - The candidate with a more than half the votes should be the winner.
• Majority candidate
  - The candidate with the majority of 1st place votes.

Criterion is an outcome that should happen but doesn’t always depend on the method.
Criterion for selecting a Winner
(Things that "should" be true about an election)

**The Majority Criterion**
If candidate X has a *majority* of the 1st place votes, then candidate X should be the *winner* of the election.

![Ballots showing majority criterion](image)

Criterion for selecting a Winner
(Things that "should" be true about an election)

**The Condorcet Criterion**
If candidate X is preferred by the voters over each of the other candidates in a head-to-head comparison, then candidate X should be the winner of the election.

![Ballots showing Condorcet criterion](image)

http://en.wikipedia.org/wiki/Marquis_de_Condorcet
Heisman Trophy Winner Selection
Alternate Voting Methods for Presidential Primaries
Results of Bush, Gore, Nader Presidential Vote in 2000
Wikipedia Article on Voting Methods and Criteria
Monotonicity Criterion
Wikipedia Voting Systems Page
wikipedia Arrows Impossibility Theorem
Wikipedia Page on Kenneth Arrow
Nice Web Page to Compare Several Types of Voting Methods