The Method of Pairwise Comparisons

In a pairwise comparison between $X$ and $Y$, every vote is assigned to either $X$ or $Y$, the vote going to whichever of the two candidates is listed higher on the ballot. The winner is the one with the most votes; if the two candidates split the votes equally, it ends in a tie.

<table>
<thead>
<tr>
<th>Number of votes</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>2nd choice</td>
<td>B</td>
<td>A</td>
<td>C</td>
<td>D</td>
</tr>
<tr>
<td>3rd choice</td>
<td>C</td>
<td>D</td>
<td>A</td>
<td>B</td>
</tr>
<tr>
<td>4th choice</td>
<td>D</td>
<td>C</td>
<td>B</td>
<td>A</td>
</tr>
</tbody>
</table>

Final Tally: A-3, B-2.5, C-2, D-1.5, E-1. (Choice A loses to B and beats C, D, and E) A wins.

How Many Pairwise Comparisons?

In an election between 5 candidates, there were 10 pairwise comparisons.

If 6 candidates, how many comparisons?

We could also count as an $C_n^2$ problem. How?
So what is wrong with the method of pairwise comparisons?

The Independence-of-Irrelevant-Alternatives Criterion (IIA)

If candidate \( X \) is a winner of an election and in a recount one of the non-winning candidates is removed from the ballots, then \( X \) should still be a winner of the election.

<table>
<thead>
<tr>
<th>Number of voters</th>
<th>2</th>
<th>6</th>
<th>4</th>
<th>1</th>
<th>1</th>
<th>4</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st choice</td>
<td>A</td>
<td>B</td>
<td>B</td>
<td>D</td>
<td>E</td>
<td>D</td>
<td>E</td>
</tr>
<tr>
<td>2nd choice</td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>B</td>
<td>D</td>
<td>A</td>
<td>C</td>
</tr>
<tr>
<td>3rd choice</td>
<td></td>
<td>D</td>
<td>A</td>
<td>A</td>
<td>E</td>
<td>D</td>
<td></td>
</tr>
<tr>
<td>4th choice</td>
<td>B</td>
<td>D</td>
<td>E</td>
<td>D</td>
<td>B</td>
<td></td>
<td>B</td>
</tr>
<tr>
<td>5th choice</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>E</td>
<td>B</td>
<td>A</td>
<td></td>
</tr>
</tbody>
</table>

Eliminate C (an irrelevant alternative) from this election and B wins (rather than A).

Do all pairwise comparisons of \( ABE \) and \( D \) and \( E \).

B wins.

Methods of Vote Counting

- Plurality
- Borda Count
- Plurality with Elimination
- Pairwise Comparisons
- Others

Fairness Criteria

- Majority Criterion
- Condorcet Criterion
- Monotonicity Criterion
- Independence of Irrelevant Alternatives Criterion
- Others

Arrow's Impossibility Theorem

It is mathematically impossible for a democratic voting method to satisfy all of the fairness criteria (in every possible case, when there are three or more candidates).

Wikipedia Voting Systems Page

Wikipedia Arrow's Impossibility Theorem

Wikipedia Page on Kenneth Arrow
Attachments

- Heisman Trophy Winner Selection
- Alternate Voting Methods for Presidential Primaries
- Results of Bush, Gore, Nader Presidential Vote in 2000
- Wikipedia Article on Voting Methods and Criteria
- Monotonicity Criterion
- Wikipedia Voting Systems Page
- wikipedia Arrows Impossibility Theorem
- Wikipedia Page on Kenneth Aarow
- Nice Web Page to Compare Several Types of Voting Methods