

Another Example from the Book:

Table 4-3 Republic of Parador (Population by State)

Assign a number of seats in Congress to each of the following 6 states in proportion to their relative populations. There are 250 seats in the congress.

Find the Standard Quotient (Population per Seat)

$$\frac{\text{Total Population}}{\text{\# of seats}} = \frac{12,500,000 \text{ people}}{250 \text{ seats}} = 50,000$$

people per seat in congress

Make a guess the apportionment. Does it work?

State	A	B	C	D	E	F	Total
Population	1,646,000	6,936,000	154,000	2,091,000	686,000	988,000	12,500,000

Quotas $32.9 \quad 137.8 \quad \frac{154000}{50000} = 3.08 \quad 41.82 \quad 13.7 \quad 19.76$

Round off $33 \quad 138 \quad 3 \quad 42 \quad 14 \quad 20 = 251$

too many seats
How should you round?

Hamilton's Method

- Step 1. Calculate each state's standard quota.
- Step 2. Give to each state its *lower quota*.
- Step 3. Give the surplus seats to the state with the largest fractional parts until there are no more surplus seats.

Hamilton's Method of Apportionment

<http://www.cut-the-knot.org/Curriculum/SocialScience/AHamilton.shtml>



U.S. Constitution on Apportionment
http://www.archives.gov/exhibits/charters/constitution_transcript.html

Article. I.

Section. 1.

All legislative Powers herein granted shall be vested in a Congress of the United States, which shall consist of a Senate and House of Representatives.

Section. 2.

The House of Representatives shall be composed of Members chosen every second Year by the People of the several States, and the Electors in each State shall have the Qualifications requisite for Electors of the most numerous Branch of the State Legislature.

No Person shall be a Representative who shall not have attained to the Age of twenty five Years, and been seven Years a Citizen of the United States, and who shall not, when elected, be an Inhabitant of that State in which he shall be chosen.

Representatives and direct Taxes shall be apportioned among the several States which may be included within this Union, according to their respective Numbers, which shall be determined by adding to the whole Number of free Persons, including those bound to Service for a Term of Years, and excluding Indians not taxed, three fifths of all other Persons. The actual Enumeration shall be made within three Years after the first Meeting of the Congress of the United States, and within every subsequent Term of ten Years, in such Manner as they shall by Law direct. The Number of Representatives shall not exceed one for every thirty Thousand, but each State shall have at Least one Representative; and until such enumeration shall be made, the State of New Hampshire shall be entitled to chuse three, Massachusetts eight, Rhode-Island and Providence Plantations one, Connecticut five, New-York six, New Jersey four, Pennsylvania eight, Delaware one, Maryland six, Virginia ten, North Carolina five, South Carolina five, and Georgia three.

When vacancies happen in the Representation from any State, the Executive Authority thereof shall issue Writs of Election to fill such Vacancies.

The House of Representatives shall chuse their Speaker and other Officers; and shall have the sole Power of Impeachment.

Hamilton's Method worked out for our 6-state Congress Example

Each seat = 50,000 people

State	Population	Step1 Quota	Step 2 Lower Quota	Fractional parts	Step 3 Surplus	Hamilton apportionment
A	1,646,000	32.92	32	0.92	First	33
B	6,936,000	138.72	138	0.72	Last	139
C	154,000	3.08	3	0.08		3
D	2,091,000	41.82	41	0.82	Second	42
E	685,000	13.70	13	0.70		13
F	988,000	19.76	19	0.76	Third	20
Total	12,500,000	250.00	246	4.00	4	250

4 seats remaining

Rules that apportionments should follow:

The Quota Rule

No state should be apportioned a number of seats smaller than its lower quota or larger than its upper quota.

When a state is apportioned a number smaller than its lower quota, we call it a lower-quota violation;

when a state is apportioned a number larger than its upper quota, we call it an upper-quota violation.)

The most serious (in fact, the fatal) flaw of Hamilton's method is commonly known as the **Alabama paradox**.

In essence, the paradox occurs when an increase in the total number of seats being apportioned, in and of itself, forces a state to lose one of its seats.

After the 1880 census, C. W. Seaton, chief clerk of the United States Census Bureau, computed apportionments for all House sizes between 275 and 350, and discovered that Alabama would get 8 seats with a House size of 299 but only 7 with a House size of 300.

State	Size	With 10 seats		With 11 seats	
		Fair share	Seats	Fair share	Seats
A	6	4.286	4	4.714	5
B	6	4.286	4	4.714	5
C	2	1.429	2	1.571	1

SD standard division
 $\# \text{ people / seat} = \frac{14}{10 \text{ seats}} = 1.4$
 $6 / 1.4 = 4.286$ seats is my Quota.
 extra seats to give out go to A & B.

Wikipedia Alabama Paradox

The Hamilton's method can fall victim to two other paradoxes called

The **population paradox**- when state A loses a seat to state B even though the population of A grew at a higher rate than the population of B.

TABLE 4-10 Intergalactic Congress: Apportionment of 2525

Planet	Population	Step 1	Step 2	Step 3	Apportionment
Alanos	150	8.3	8	0	8
Betta	78	4.3	4	0	4
Conii	173	9.61	9	1	10
Dugos	204	11.3	11	0	11
Ellisium	295	16.38	16	1	17
Total	900	50.00	48	2	50

Happened to Ohio in last census.

Quota round down odd back in # of seats total

TABLE 4-11 Intergalactic Congress: Apportionment of 2535

Planet	Population	Step 1	Step 2	Step 3	Apportionment
Alanos	150	8.25	8	0	8
Betta	78	4.29	4	1	5
Conii	181	9.96	9	1	10
Dugos	204	11.22	11	0	11
Ellisium	296	16.28	16	0	16
Total	909	50.00	48	2	50

← same population gained a seat

← increased population lost a seat

The **new-states paradox**- that the addition of a new state with its fair share of seats can, in and of itself, affect the apportionments of other states.

TABLE 4-12 Metro Garbage Truck Apportionments

District	Homes serviced	Quota (SD = 1000)	Hamilton apportionment
Northtown	10,450	10.45	10
Southtown	89,550	89.55	90
Total	100,000	100.00	100

100 trucks for North + South

TABLE 4-13 Revised Metro Garbage Truck Apportionments

District	Homes serviced	Quota (SD ≈ 1002.38)	Hamilton apportionment
Northtown	10,450	10.42	11
Southtown	89,550	89.34	89
Newtown	5,250	5.24	5
Total	105,250	105.00	105

← down 1 truck.

Add a new "state" and add enough trucks for them.

Changes the apportionment of other states.

Jefferson's Method

Step 1. Find a "suitable" divisor D.

A suitable or **modified divisor** is a divisor that produces an apportionment of exactly M seats when the quotas (populations divided by D) are *rounded down*.

Step 2. Each state is apportioned its *lower quota*.







OLD
50000 people/seat
New 49000 people/seat
Higher Quotas
So when you round down
you get 250 seats.

Bad News- Jefferson's method can produce **upper-quota violations!**

To make matters worse, the upper-quota violations tend to consistently favor the larger states.

The apportionment method suggested by Alexander Hamilton was approved by Congress in 1791, but was subsequently vetoed by president Washington - in the very first exercise of the veto power by President of the United States. Hamilton's method was adopted by the US Congress in 1852 and was in use through 1911 when it was replaced by Webster's method.

Attachments

-  [Hamilton's Method of Apportionment](#)
-  [Section 2 of Constitution: Apportionment of Representatives](#)
-  [Wikipedia Alabama Paradox](#)
-  [Webster's Method Finding Suitable Divisor](#)
-  [Projected Changes in Representatives 2010 Census](#)
-  [State Populations as of 2008](#)