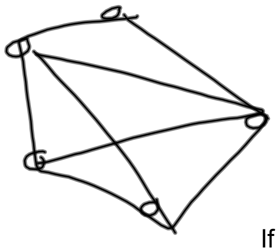
1. What is it about the degrees of the vertices of a graph that tells you whether there is an Euler circuit, or just an Euler path or neither?

If you have a graph, an Euler circuit is a circuit passes over every edge and that doesn't repeat any edges and starts and ends at the same spot.

If you have a graph, an Euler path is a path passes over every edge and that doesn't repeat any edges and starts and ends at different vertices.



If a graph has exactly 2 odd vertices, then there is an Euler path. Moreover, the path has to start at one odd vertices, and end at the other.

If a graph has all even vertices (and no odd), then there is an Euler circuit. You can start and end at any vertex.

If a graph has more than 2 odd vertices, then there is no Euler path in the graph.

The Traveling Salesman Problem

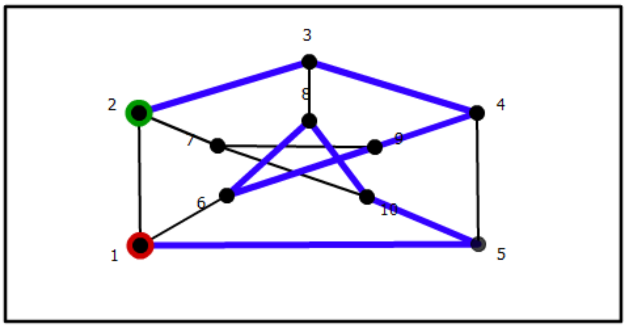
- **Hamilton path**
  A path that visits each VERTEX of the graph once and only once.
- **Hamilton circuit**
  A circuit that visits each VERTEX of the graph once and only once (at the end, of course, the circuit must return to the starting vertex).

- **Euler path**
  A path that visits each EDGE of the graph once and only once.
- **Euler circuit**
  A circuit that visits each EDGE of the graph once and only once (at the end, of course, the circuit must return to the starting vertex).

http://en.wikipedia.org/wiki/William_Rowan_Hamilton

http://wps.prenhall.com/esm_tannenbaum_excursions_5/14/3687/943975.cw/index.html

http://www.flashandmath.com/mathlets/discrete/graphtheory/hamiltongraphs.html

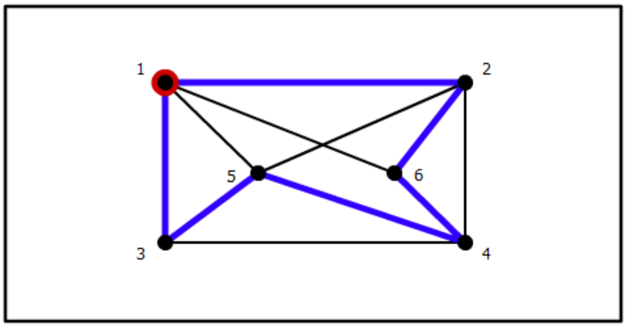


Seem to get Stuck here

Petersen Graph.

No Hamilton Circuit

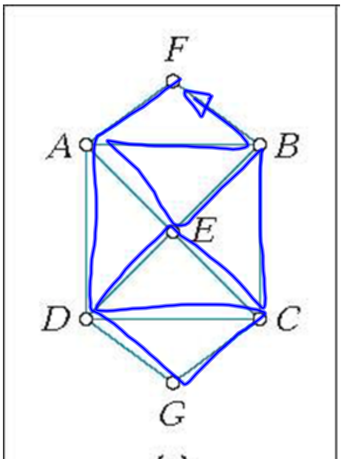
Does have Hamilton Path.



Does have a Hamilton Circuit

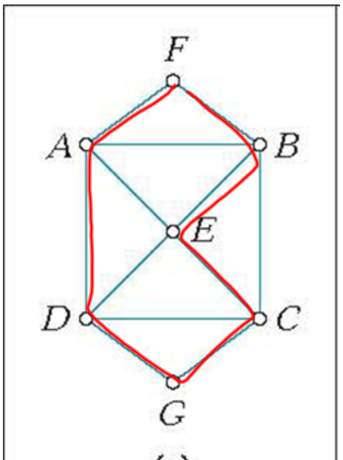
Do the following graphs have
Euler Paths? Euler Circuits? Hamilton Paths? Hamilton Circuits?

If not, why not? If so, list them.



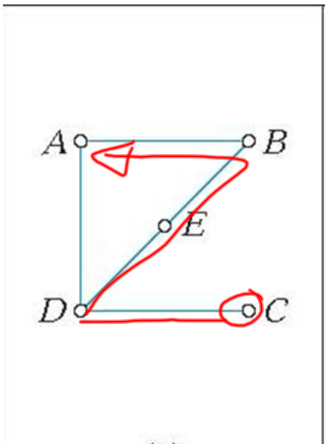
(a)

Euler Circuit



(a)

Hamilton Circuit

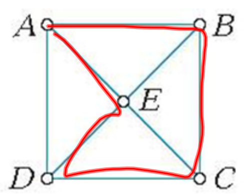


(b)

Hamilton Path
cant get back to start.

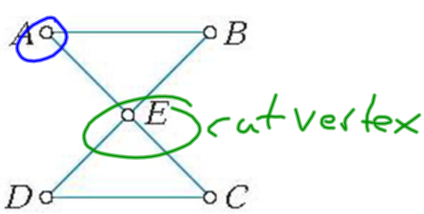
Do the following graphs have
Euler Paths? Euler Circuits? Hamilton Paths? Hamilton Circuits?

If not, why not? If so, list them.



Yes, many Hamilton circuits

(c)



No. "center" messes it up.

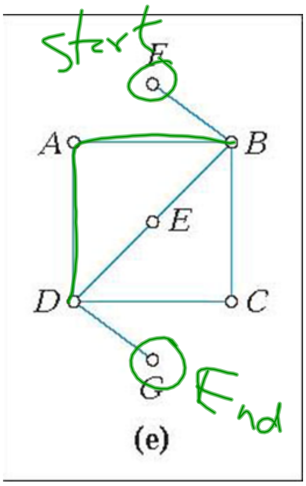
(d)

You have to go through 'E' Twice

'E' is a cut-vertex.

Do the following graphs have
Euler Paths? Euler Circuits? Hamilton Paths? Hamilton Circuits?

If not, why not? If so, list them.



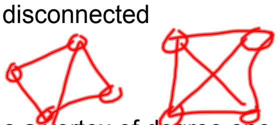
(e)

No Hamilton Circuit
since degree 1 vertex

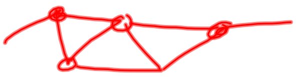
Is there a Hamilton Path?
No

Reasons why a graph might not have a Hamilton Circuit:

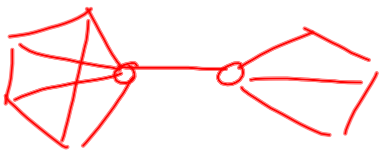
1. If the graph is disconnected



2. If the graph has a vertex of degree one.



3. If the graph has an edge that is a bridge.



No possible circuit through every city exactly once.

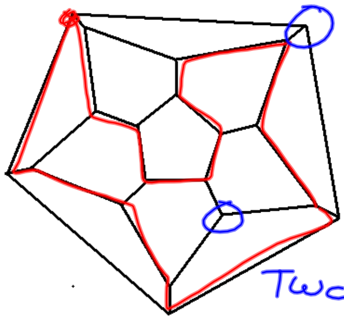
But there is no "nice" reason that explains when a graph has no Hamilton Circuit.

The only reason a connected graph has no Euler circuit is that it has odd vertices.

Examples of Graphs: Do they have a Hamilton Circuit?

20 vertices

Icosahedron

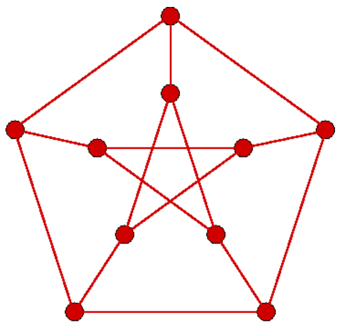


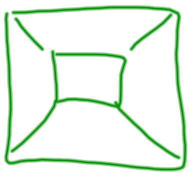
missed one

12-sided die

Two.... Can be done.. Try it.

The Petersen Graph

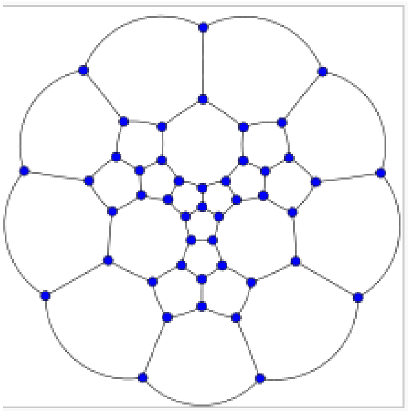




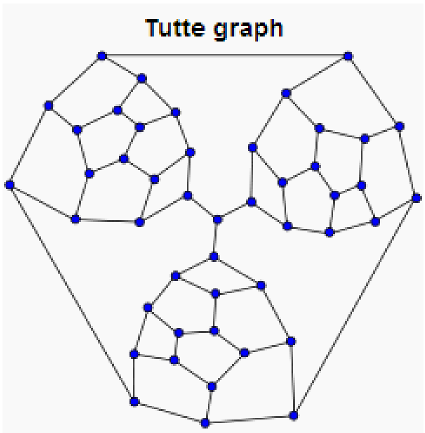
6-sided die "Cube"

http://www.flashandmath.com/mathlets/discrete/graphtheory/graph2.html

Two nice planar Non-Hamiltonian Graphs



A graph that can be proven non-Hamiltonian using Grinberg's theorem



Tutte graph

http://en.wikipedia.org/wiki/W._T._Tutte

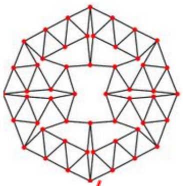
NP-Complete Problems

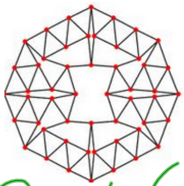
There are many famous algorithm questions (some about graphs, some about various other math problems) that are so-called NP-Complete.

Nobody knows a "simple, quick" way to answer them without more or less trying every possible case. Like finding a hamilton circuit. However, if someone shows you a solution, it is easy to see that it works. Again like a Hamilton circuit.

Suppose somebody gives you a big graph (say 60 vertices). What do you have to check to see it it has an

Euler Circuit? Hamilton Circuit?





Every vertex degree 4
So Yes there is Euler Circuit

Computer (or person) has to check every possible path.

3 choices of outbound edge at each vertex
⇒ about 3^{60} possible circuits
≈ 5000000000000000000000000000 possible circuits

http://en.wikipedia.org/wiki/NP-complete

There are, however, nice theorems that identify special situations where a graph must have a Hamilton circuit.

For instance, the previous graph must be Hamiltonian by:
Tutte's Theorem : Any 4-regular planar graph has a Hamiltonian circuit.

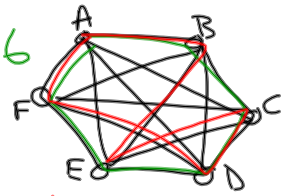
Another well-known known theorems is:
Dirac's theorem:
If a connected graph has N vertices (N > 2) and all of them have degree bigger or equal to N / 2, then the graph has a Hamilton circuit.

But nothing is known to work for all graphs to decide if it has a Hamilton circuit or not, other than checking all possible circuits.

If a graph has a Hamilton circuit, then how many different Hamilton circuits does a it have?

A graph with N vertices in which *every* pair of distinct vertices is joined by an edge is called a **complete graph** on N vertices and denoted by the symbol K_N.

K6



6 cities and all edges connecting them

Circuit ABCDEF

or ABEDCF

How many possible Hamilton circuits?

$N = 6\cdot5\cdot4\cdot3\cdot2\cdot1 = 6!$

6 different starting points
these all are the same final circuit

ABCDEF
BCDEFA
CDEFAB
D . . . .
E . . . .
F . . . .

So Divide by 6

$N = \frac{6\cdot5\cdot4\cdot3\cdot2\cdot1}{6} = 5!$

How many edges are there in a complete graph (no loops or multiple edges) with "n" vertices?

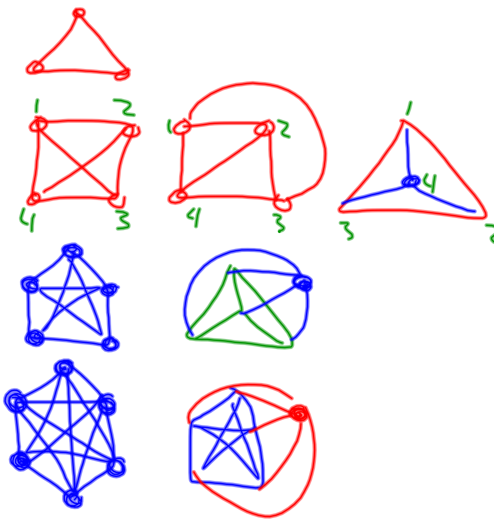
n=3

n=4

n=5

n=6

n



# Edges	# circuits
3	2! = 2
6 (3+3)	3! = 6
10 (6+4)	4! = 24
15 (10+5)	5! = 120

15 + 6 more = 21 edges

n=7

$_7C_2 = \frac{7!}{(5!)\,2!} = \frac{7\cdot6}{2\cdot1} = 21$