Midterm 2 Review Guide

Definitions & Terms: Write your own careful and precise definitions

Graph
Vertex
Edge
Path
Connected
Bridge
Circuit
Tree
Forest
Degree of a vertex

Euler Circuit
Euler Path
Hamilton Circuit
Spanning Tree
Eulerize a Graph
Complete Graph
Algorithm

Efficient vs. Nonefficient

Optimal vs NonOptimal

Theorems about Euler Circuits and Euler Paths

If _______ then there is an Euler Circuit.

If _______ then " " " Path

If _______ then neither is possible

Examples:
Eulerize a Circuit:

Why would you want to do this? Can you think of a practical example?
How do you do it? Which edges do you add? What do you end up with when you are done?
Algorithms for Hamilton Circuits (List a few and describe them)
Algorithms for Minimum Spanning Trees

List a few and describe them

Hamilton Circuits Section

11. For the graph shown in Fig. 6-30,
   (a) find a Hamilton path that starts at $A$ and ends at $D$.
   (b) find a Hamilton path that starts at $G$ and ends at $H$.
   (c) explain why the graph has no Hamilton path that starts at $B$.
   (d) explain why the graph has no Hamilton circuit.

   ![Diagram](image)
31. For the weighted graph shown in Fig. 6-38, (i) find the indicated tour, and (ii) give its cost. (Note: This is the graph discussed in Example 5.7.)
   (a) The nearest-neighbor tour with starting vertex B
   (b) The nearest-neighbor tour with starting vertex C
   (c) The nearest-neighbor tour with starting vertex D
   (d) The nearest-neighbor tour with starting vertex E

![Graph Image]

**FIGURE 6-38**

Spanning Trees Section

In Exercises 5 through 8, assume that $G$ is a graph with no loops or multiple edges, and choose the option that best applies: (I) $G$ is definitely a tree (explain why); (II) $G$ is definitely not a tree (explain why); or (III) $G$ may or may not be a tree (in this case, give two examples of graphs that fit the description—one a tree and the other one not).

6. (a) $G$ has 10 vertices and 11 edges and is a connected graph.
   (b) $G$ has 10 vertices and 9 edges.
   (c) $G$ has 10 vertices and for some pair of vertices $X$ and $Y$ in $G$ there are two paths from $X$ to $Y$. 
(d) $G$ has 10 vertices and for every pair of vertices $X$ and $Y$ in $G$ there is at least one path from $X$ to $Y$.

(e) $G$ has 10 vertices and for every pair of vertices $X$ and $Y$ in $G$ there is exactly one path from $X$ to $Y$.

8. (a) $G$ has 10 vertices, and there is a Hamilton circuit in $G$.

(b) $G$ is connected and has 10 vertices. Every vertex has degree 9.

(c) $G$ is connected and has 10 vertices. One of the vertices has degree 9, and all other vertices have degree less than 9.

(d) $G$ is connected and has 10 vertices, and every vertex has degree 2.

10. (a) Find all the spanning trees of the network shown in Fig. 7-36(a).

(b) Find all the spanning trees of the network shown in Fig. 7-36(b).

(c) How many different spanning trees does the network shown in Fig. 7-36(c) have?
22. For the network shown in Fig. 7-44,
   (a) find the MST of the network using Kruskal's algorithm.
   (b) give the weight of the MST found in (a).

Is there more than one MST for this graph?
How is that possible?
Are they all "optimal"?

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**FIGURE 7-44**

41. Give an example of a graph with $N = 11$ vertices and $M = 10$ edges having
   (a) exactly one circuit.
   (b) exactly two circuits.
   (c) exactly three circuits.
JOGGING

51. (a) How many spanning trees does the network shown in Fig. 7-65(a) have?
(b) How many different spanning trees does the network shown in Fig. 7-65(b) have?
(c) How many different spanning trees does the network shown in Fig. 7-65(c) have?