

Math 1148  
Autumn 2012  
College Algebra

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Course Web Page

<http://www.math.osu.edu/~maharry.1/1148Fall2012/Math1148Autumn2012.html>

Suggested Reading:

Skim through Chapter 1. Make sure all the concepts are familiar. Ask if they are not.  
Take a careful look at the Chap. 1 test on page 128. Make sure that you can do those problems.

Read the Principles of Problem Solving Page P1. It gives some general guidelines for problem solving that can be used in this course and in many others.

### Sec. 1.6 Modeling With Equations: Solving Word Problems

**22. Investments** If Ben invests \$4000 at 4% interest per year, how much additional money must he invest at  $5\frac{1}{2}\%$  annual interest to ensure that the interest he receives each year is  $4\frac{1}{2}\%$  of the total amount invested?

$$\begin{array}{l} \text{interest} \\ \text{from 1st} \\ \text{account} \end{array} + \begin{array}{l} \text{interest} \\ \text{in 2nd} \\ \text{account} \end{array} = \text{Total interest}$$

$X = \text{amount in 2nd account.}$

$$4000(.04) + X(.055) = (4000 + X)(.045)$$

Total amount      Overall interest

How to solve it?

$$\begin{aligned} 160 + .055X &= 180 + (.045)X \\ (.010X) &= 20 \\ X &= 2000 \text{ dollars.} \end{aligned}$$

27. **Inheritance** Craig is saving to buy a vacation home. He inherits some money from a wealthy uncle, then combines this with the \$22,000 he has already saved and doubles the total in a lucky investment. He ends up with \$134,000—just enough to buy a cabin on the lake. How much did he inherit?

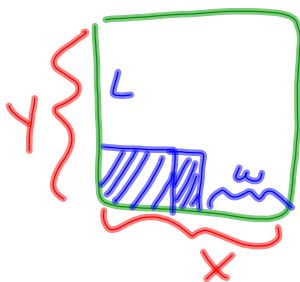
Variable =  $w$  = starting money

$$2 \cdot (w + 22000) = 134000$$

$$w + 22000 = 67,000$$

$$w = 45000$$

37. **Dimensions of a Lot** A square plot of land has a building 60 ft long and 40 ft wide at one corner. The rest of the land outside the building forms a parking lot. If the parking lot has area 12,000 ft<sup>2</sup>, what are the dimensions of the entire plot of land?



Variables  $X$  and  $Y$

Square so  $X = Y$

$$\text{Area of plot} - \text{Area Building} = \text{Area of Parking Lot}$$

$$X^2 - (60)(40) = 12000$$

$$X^2 = 14400$$

$$X = \pm 120$$

$$X = 120 \text{ ft}$$

55. **Mixture Problem** A jeweler has five rings, each weighing 18 g, made of an alloy of 10% silver and 90% gold. She decides to melt down the rings and add enough silver to reduce the gold content to 75%. How much silver should she add?

$$\begin{array}{|c|} \hline .90 \text{ gold} \\ \hline .10 \text{ silver} \\ \hline \end{array} + \begin{array}{|c|} \hline 100\% \\ \hline \text{Silver} \\ \hline \end{array} = \begin{array}{|c|} \hline .75 \text{ gold} \\ \hline .25 \text{ silver} \\ \hline \end{array}$$

$$(.10)(90 \text{ g}) + (1.00)(X) = (.25)(90 + X)$$

(% silver)(amount)

$X = \text{grams of silver}$

Solve

$$(.1)(9) + (1)(X) = (.25)(90 + X)$$

$$9 + X = 22.5 + .25X$$

$$.75X = 13.5$$

$$\frac{3}{4}X = \frac{27}{2}$$

$$X = \frac{27}{2} \cdot \frac{4}{3} = \frac{54}{3} = 18 \text{ grams}$$

64. **Sharing a Job** Next-door neighbors Bob and Jim use hoses from both houses to fill Bob's swimming pool. They know that it takes 18 h using both hoses. They also know that Bob's hose, used alone, takes 20% less time than Jim's hose alone. How much time is required to fill the pool by each hose alone?

73. **Distance, Speed, and Time** It took a crew 2 h 40 min to row 6 km upstream and back again. If the rate of flow of the stream was 3 km/h, what was the rowing speed of the crew in still water?

## Sec. 1.7 Inequalities

RULES FOR INEQUALITIES	
Rule	Description
1. $A \leq B \Leftrightarrow A + C \leq B + C$	<b>Adding</b> the same quantity to each side of an inequality gives an equivalent inequality.
2. $A \leq B \Leftrightarrow A - C \leq B - C$	<b>Subtracting</b> the same quantity from each side of an inequality gives an equivalent inequality.
3. If $C > 0$ , then $A \leq B \Leftrightarrow CA \leq CB$	<b>Multiplying</b> each side of an inequality by the same <i>positive</i> quantity gives an equivalent inequality.
4. If $C < 0$ , then $A \leq B \Leftrightarrow CA \geq CB$	<b>Multiplying</b> each side of an inequality by the same <i>negative</i> quantity <i>reverses the direction</i> of the inequality.
5. If $A > 0$ and $B > 0$ , then $A \leq B \Leftrightarrow \frac{1}{A} \geq \frac{1}{B}$	<b>Taking reciprocals</b> of each side of an inequality involving <i>positive</i> quantities <i>reverses the direction</i> of the inequality.
6. If $A \leq B$ and $C \leq D$ , then $A + C \leq B + D$	Inequalities can be added.

$$3 < 5$$

$$-2(3) \quad -2(5)$$

$$-6 > -10$$

### GUIDELINES FOR SOLVING NONLINEAR INEQUALITIES

- 1. Move All Terms to One Side.** If necessary, rewrite the inequality so that all nonzero terms appear on one side of the inequality sign. If the nonzero side of the inequality involves quotients, bring them to a common denominator.
- 2. Factor.** Factor the nonzero side of the inequality.
- 3. Find the Intervals.** Determine the values for which each factor is zero. These numbers will divide the real line into intervals. List the intervals that are determined by these numbers.
- 4. Make a Table or Diagram.** Use test values to make a table or diagram of the signs of each factor on each interval. In the last row of the table determine the sign of the product (or quotient) of these factors.
- 5. Solve.** Determine the solution of the inequality from the last row of the sign table. Be sure to check whether the inequality is satisfied by some or all of the endpoints of the intervals. (This may happen if the inequality involves  $\leq$  or  $\geq$ .)

25.  $4 - 3x \leq -(1 + 8x)$

26.  $2(7x - 3) \leq 12x + 16$

$$4 - 3x \leq -1 - 8x$$

$$5 \leq -5x$$

$$-1 \geq x \quad \text{switch } \leq \text{ to } \geq$$

$$\text{or}$$

$$x \leq -1$$



$$(-\infty, -1]$$

107. **Car Rental Cost** A car rental company offers two plans for renting a car.

Plan A: \$30 per day and 10¢ per mile

Plan B: \$50 per day with free unlimited mileage

For what range of miles will Plan B save you money?

Ans Budget

Assume only 1 day  
 what about if rent for a week?  
 or Y days?

Variable =  $X$  miles (10¢) mile

Solve  $50 < 30 + (.10)(x)$

Plan B                  Plan A

$20 < (.1)x$

$200 < x$

more than 200, Plan B cheaper

Weeklong Vacation

$$50(7) < 30(7) + .10(x)$$

$$350 < 210 + .10(x)$$

$$140 < .10(x)$$

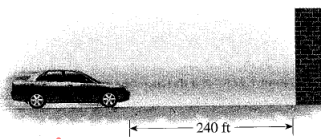
$1400 < x$

more than 1400 miles, Plan B cheaper

117. **Stopping Distance** For a certain model of car the distance  $d$  required to stop the vehicle if it is traveling at  $v$  mi/h is given by the formula

$$d = v + \frac{v^2}{20}$$

where  $d$  is measured in feet. Kerry wants her stopping distance not to exceed 240 ft. At what range of speeds can she travel?



Find a range for  $V =$  velocity.

Want  $d < 240$

$$v + \frac{v^2}{20} < 240$$

(mult by 20)

$$20v + v^2 < 4800$$

$$v^2 + 20v - 4800 < 0$$

$$(v + 80)(v - 60) < 0$$

$$v = -80 \quad v = 60$$

Not practical

Want  $V < 60$

positive = 0      negative      = 0      positive

←----- -80      0      60 ----->

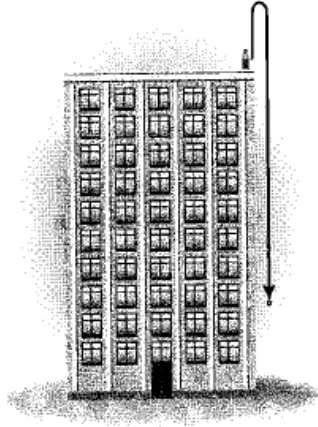
(neg)(neg)      (neg)(pos)      (pos)(pos)

$-80 < V < 60$

**115. Falling Ball** Using calculus, it can be shown that if a ball is thrown upward with an initial velocity of 16 ft/s from the top of a building 128 ft high, then its height  $h$  above the ground  $t$  seconds later will be

$$h = 128 + 16t - 16t^2$$

During what time interval will the ball be at least 32 ft above the ground?



What happens when you have quadratic terms in the inequality?

41.  $2x^2 + x \geq 1$

42.  $x^2 < x + 2$

move to one side

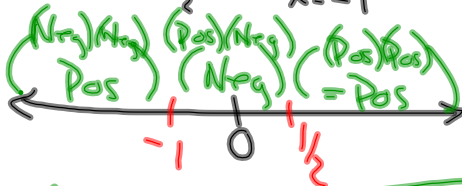
$$2x^2 + x - 1 \geq 0$$

quadratic formula  
or

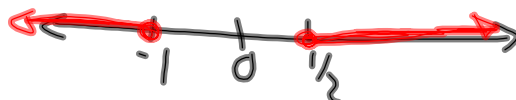
$$(2x - 1)(x + 1) \geq 0$$

$$2x - 1 = 0 \quad x + 1 = 0$$

$$x = \frac{1}{2} \quad x = -1$$

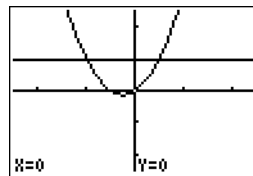


Answer =  $x \leq -1$  or  $x \geq \frac{1}{2}$   $(-\infty, -1] \cup [\frac{1}{2}, \infty)$



$$2(x^2 + x - 1) \geq 0$$

Not the same



$$50. (x - 5)(x - 2)(x + 1) > 0$$

$$57. \frac{x - 3}{x + 1} \geq 0$$

$$58. \frac{2x + 6}{x - 2} < 0$$



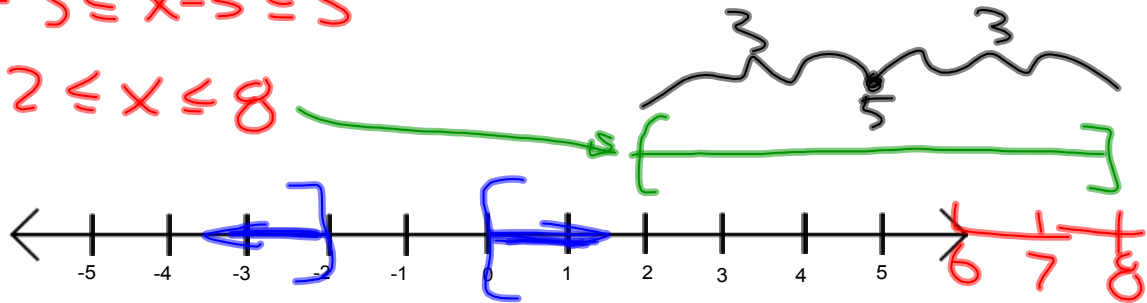
How do you handle Absolute Value Signs?

77.  $|x - 5| \leq 3$

78.  $|x + 1| \geq 1$

$-3 \leq x - 5 \leq 3$

$2 \leq x \leq 8$

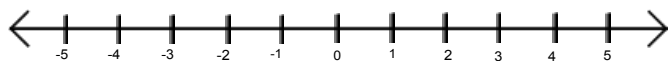
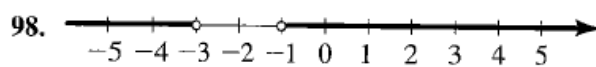
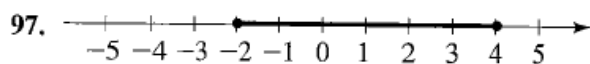
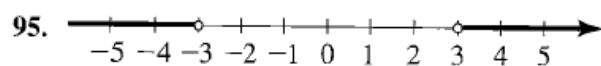
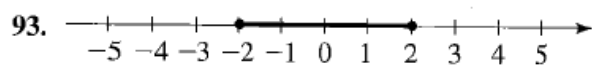


$|x + 1| \geq 1$   
 $|x - (-1)| \geq 1$  Distance from  $-1$  is at least 1

81.  $|3x - 2| \geq 5$

82.  $|8x + 3| > 12$

93–98 ■ A set of real numbers is graphed. Find an inequality involving an absolute value that describes the set.



### Quiz Problem

1. Suppose you want to build a fence around a garden to keep out the deer. You go to Home Depot and buy 120 feet of flexible fencing. You want to build the garden in the shape of a rectangle with fence on all four sides and need it to be at least 800 sq ft in area. Set up an inequality and solve it to find the range of possible lengths for your garden.

Area  $\geq 800$

$L \cdot w \geq 800$

$2L + 2w = 120$

$2(L + w) = 120$

$L + w = 60$

Solve for a variable  $\rightarrow L = 60 - w$

$(60 - w) \cdot w \geq 800$

$60w - w^2 \geq 800$

or  $0 \geq w^2 - 60w + 800$

$0 \geq (w - 20)(w - 40)$





$w = 20$  or  $w = 40$

positive (Neg) (pos)

Answer  $20 \leq x \leq 40$

## Attachments

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-  Trig Applets
-  Multigrapher
-  VisualFunction
-  Function as a table