Sec. 4.4 Laws of Logarithms

There are 3 basic laws for logarithms and they each come from the three basic laws for combining exponents.

$Log_a(A \cdot B) =$ $a^x \cdot a^y$

$Log_a\left(\frac{A}{B}\right) =$ $\frac{a^x}{a^y}$

$Log_a\left(A^C\right) =$ $\left(a^x\right)^y$

Sec. 4.4 Laws of Logarithms

These are the common mistakes with Logarithms. Watch for these.

There is often not much you can do to simplify expressions like these (either the left side or the right).

$$\log_a(A+B) \neq \log_a(A) \cdot \log_a(B)$$

$$\left(\frac{\log_a A}{\log_a B}\right) \neq \log_a(A) - \log_a(B)$$

$$\left(\log_a(A)\right)^C \neq C\log_a(A)$$

Examples of Simplifying: Break apart into several simple log functions. Evaluate if you can.

$$Log\left(\frac{10^4 \cdot \sqrt{1000}}{10^3 \cdot \frac{1}{10}}\right)$$

Examples of Simplifying: Simplify the exponentials first.

$$Log\left(\frac{10^4 \cdot \sqrt{1000}}{10^3 \cdot \frac{1}{10}}\right)$$

$$Log\left(\frac{(x+1)^3 \cdot \sqrt{x-3}}{(x-2)^4}\right)$$

Examples of Simplifying: Combine into a single (more complicated) log function.

$$3\log(x+1)+2\log(x)-\log(x-3)=$$

$$\frac{1}{2}\ln(x)-3\ln(x^2)+5\ln(x+1)=$$

Change of Base Formula

It is often useful to change a logarithm to either base ’10’ or base ‘e’.

To change $\log_b(x)$ into $\log_a(x)$

Consider the two similar equations with different bases.

$$P(x) = 5\left(\sqrt{10}\right)^x \qquad P(t) = 5(10)^t$$

Solve for when they P=500

Change of Base Formula

It is often useful to change a logarithm to either base ’10’ or base ‘e’.

To change $\log_b(x)$ into $\log_a(x)$

Start with

$$b^y = x \qquad y = \log_b(x)$$

Take log base ‘a’ of both sides

$$\log_a\left(b^y\right) = \log_a(x)$$

Solve for y

$$y\log_a(b) = \log_a(x)$$

$$y = \frac{\log_a(x)}{\log_a(b)}$$

Using Ln and Log to solve equations where the base is not "e"or "10"

Suppose the intensity of Light is cut by half every 50 meters down in the ocean. How deep will the light be only 1% of the surface amount?

Sec. 4.5 Exponential and Logarithmic Equations

Guidelines for Solving Exponential Equations (page 359)

- Isolate the exponential expression on one side of the equation
- Take the logarithm of each side, then “bring down the exponent” (which gets the variable out of the exponent)
- Use algebra to solve for the variable. And check your answer.

Guidelines for Solving Logarithmic Equations (page 362)

- Isolate the logarithmic expression on one side of the equation (you may first need to combine the log terms)
- Raise the base to each side of the equation. (This gets rid of the log function)
- Use algebra to solve for the variable. And check your answer.

Examples

.

$$5^{3x-1} - 12 = 0$$

Examples

$$5^{x} = 4^{x+1}$$

$$5^x = 4^{x+1}$$

Examples

.

$$3500 = 1000(1.06)^x$$

Examples

.

$$\log_2(3x-1)=3$$

Examples

.

$$5+3Log(4x)=17$$

Examples

$$\log(x)+\log(x-1)=\log(4x)$$

$$\log(x)+\log(x-1)=\log(4x)$$

City A grows from 1000 people at 9% per year

City B grows from 2500 at only 3% per year.

$$1000(1.09)^x = 2500(1.03)^x$$

$$1000(1.09)^x = 2500(1.03)^x$$