Game Theory

Equilibrium Points (or Saddle Points) in Total conflict games (no collaborating)

Suppose Lisa and Henry are "playing a game" or "making a decision". Lisa will pay Henry the agreed upon amount after they both make their "secret" choices.

Lisa's Goal: Minimize Value
Lisa's Strategy: Minimize the Maximum Possible Value. MINIMAX

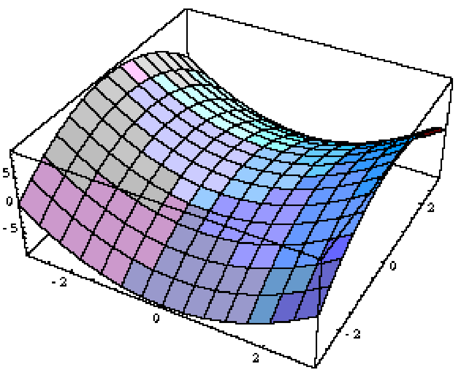
Henry's Goal: Maximize Value
Henry's Strategy: Maximize the Minimum posisble value: MAXIMIN

		Lisa Pays 1	2	3
	A	10	4	6
Henry gets	B	6	5	9
	C	2	3	7

If Lisa's MINIMAX Value is equal to Henry's MAXIMIN value, then those strategies are an equilibrium point. Or a Saddlepoint.

Neither player will gain anything if they switch their choice (and the other player stays with the same strategy)

There is no incentive to alter your strategy.



What if there is no Equilibium strategy?

		Lisa Pays		
		1	2	3
Henry gets	A	10	4	6
	B	6	~~5~~ 9	~~9~~ 5
	C	2	3	7

Lisa's Minimum of the Maximums = 7 when she picks Column 3

Henry's Maximum of Minimums = 5 when he picks Row 2.

How much does she pay him if they both play these strategies?

What should Henry do if he expects Lisa to pick Column 3?

What should Lisa do if she suspects Henry will change his pick?

A Better Idea

The play of many total-conflict games requires an element of surprise, which can be realized in practice by making use of a mixed strategy.

A **mixed strategy** is a particular randomization over a player's strategies (which henceforth we call **pure strategies**—the definite options a player can choose—to distinguish them from mixed strategies). Each one of the player's pure strategies is assigned some probability, indicating the relative frequency with which the pure strategy will be played. The specific pure strategy that will be used in any given play of the game is selected by some appropriate probabilistic mechanism or random device.

Let's play a different game.

Suppose we both call out "Heads" or "Tails".
If we call different choices, then I pay you $3.
If we both call the same, you pay me. You pay me $5 if we both call heads. And $1 if we both call tails.

What do you call out? What is your strategy? Is there an equilibrium?

Mixed Strategy:

Why does it need to be random? What if I call in a pattern H,H,T,H,H,T,H,H,T....?

		Me (wants positive) H	Me (wants positive) T
You (want negative)	H	5	-3
	T	-3	1

My Mixed Strategy: MaxiMin
Suppose I decide that 100% of the time I will pick Heads.
And the "rest of the time" (0)% of the time I will pick tails.

Expected value (for Me) if you play Heads:

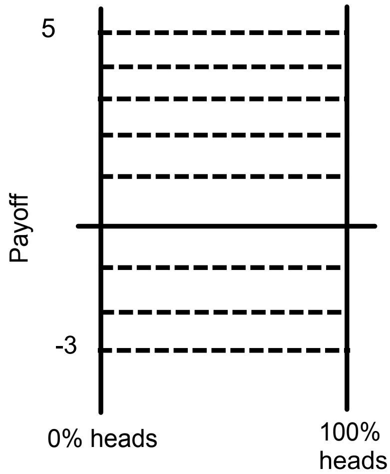
Expected value (for Me) if you play Tails:

My Mixed Strategy: MaxiMin
Suppose I decide that 0% of the time I will pick Heads.
And the "rest of the time" (100)% of the time I will pick tails.

Expected value (for Me) if you play Heads:

Expected value (for Me) if you play Tails:

		Me (wants positive) H	Me (wants positive) T
You (want negative)	H	5	-3
	T	-3	1

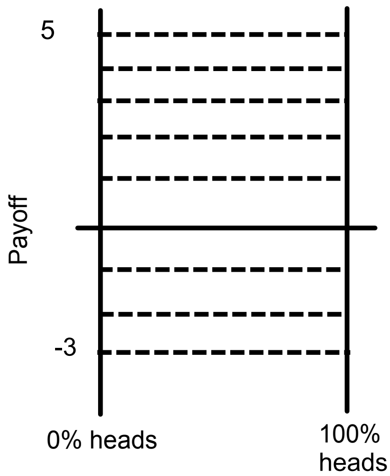


My Mixed Strategy: MaxiMin
Suppose I decide that 50% of the time I will pick Heads.
And the "rest of the time" 50% of the time I will pick tails.

Expected value (for Me) if you play Heads:

Expected value (for Me) if you play Tails:

		Me (wants positive) H	Me (wants positive) T
You (want negative)	H	5	-3
	T	-3	1

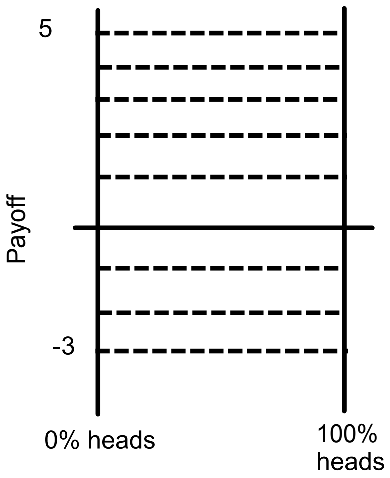


My Mixed Strategy: MaxiMin
Suppose I decide that 80% of the time I will pick Heads.
And the "rest of the time" 20% of the time I will pick tails.

Expected value (for Me) if you play Heads:

Expected value (for Me) if you play Tails:

You (want negative) \ Me (wants positive)		H	T
	H	5	-3
	T	-3	1



My Mixed Strategy: MaxiMin
Suppose I decide that P% of the time I will pick Heads.
And the "rest of the time" (100-P)% of the time I will pick tails.

Expected value (for Me) if you play Heads:

Expected value (for Me) if you play Tails:

Graph those.

I want to find the Maximum of the Minumum Expected Values. So what point do I choose?

Can we solve for the "optimal" percentage? Or the Optimal Mixed Strategy?

$$5P+(-3)(100-P) = (-3)P+(1)(100-P)$$

Solve for P

$5P-300+3P = -3P + 100 - P$

$8P -300 = 100 - 4P$

$12P = 400$

$P = 400/12 = 33.333\ \%$

What is my expected value if I play those percenages?

$5(1/3) + (-3)(2/3) =$

$5/3 - 6/3 = -1/3$

Now do it from the other players perspective. It turns out that the other player might get a different percentage of time they play heads), but the game will have the same expected value.

In our case, one player find the expected value of -1/3. i.e. they expect to lose 1/3 dollar (on average) each game

While the other player finds the expected value of -1/3 However since "negative" is the amount they pay out, they expect to gain 1/3 dollars per game.

In other words, we have an equilibrium.
The MINIMAX is equal to the MAXIMIN

In this case (since the -3's are the same, the payoff matrix is "symmetric")
the equations the other player arrives at are the same.
So the optimal probabilities are the same too.

Both choose Heads 1/3 of the time and Tails 2/3 of the time

http://www.cut-the-knot.org/Curriculum/Games/MixedStrategies.shtml

Try another example: A pitcher can either throw a Fastball or a Curveball.
The batter can be expecting either a Fastball or a Curveball.

The batter gets a hit with various percentages.
The Pitcher wants to Minimize the Maximum probability of getting a hit.
The Batter wants to Maximize the Minimum probability.

TABLE 15.8 A Baseball Duel with Probabilities

		Pitcher F	*Pitcher* C	
Batter	F	.300	.200	q
	C	.100	.500	$1-q$
		p	$1-p$	

Original Minimax strategy for the Pitcher:

Original Maximin Strategy for the Batter.

Who wants to switch strategies?

Find a Mixed Strategy for the Pitcher. Suppose he picks P% Fastballs and (1-P)% Curveballs

TABLE 15.8 A Baseball Duel with Probabilities

		Pitcher		
		F	*C*	
Batter	*F*	.300	.200	q
	C	.100	.500	$1 - q$
		p	$1 - p$	

The MINIMAX Theorem guarantees that there is a unique "game value" and an optimal strategy for each player, so that either player alone can realize at least this value by playing this strategy, which may be pure or mixed.

http://en.wikipedia.org/wiki/Minimax

16. You plan to manufacture a new product for sale next year, and you can decide to make either a small quantity, in anticipation of a poor economy and few sales, or a large quantity, hoping for brisk sales. Your expected profits are indicated in the following table.

		Economy	
		Poor	**Good**
Quantity	**Small**	$500,000	$300,000
	Large	$100,000	$900,000

If you want to avoid risk and believe that the economy is playing an optimal mixed strategy against you in a two-person zero-sum game, then what is your optimal mixed strategy and the resulting expected value? Discuss some alternative ways that you may go about making your decision.