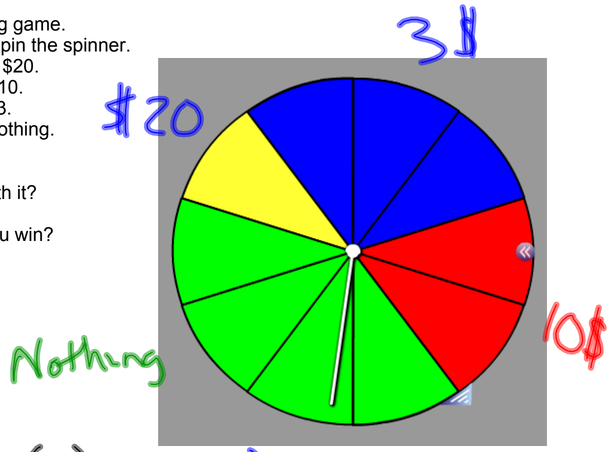
Game Theory

Expected Value:

Suppose we play the following game.
You pay me $5 to play. You spin the spinner.
If it lands on Yellow I pay you $20.
If it lands on Red I pay you $10.
If it lands on Blue I pay you $3.
If it lands on Green you get nothing.

How do you decide if it is worth it?

On average, how much do you win?



$$\frac{4}{10}(0)+\frac{1}{10}(20)+\frac{3}{10}(3)+\frac{2}{10}(10)$$

0$/spin + 2 $/spin + $\frac{9}{10}$ $/spin + 2 $/spin

Expected Value 4.9 $/spin

To find the "Expected value"

First, for each possible outcome, determine it's probability and it's "value"

The value could be "money", "years in jail", "points in a game". However you measure "value".

Find the expected value of a $2 lottery ticket.

Suppose there are three possible prizes.

$100,000 with probability 1/250,000
$10,000 with probability 1/15,000
$10 with probability 1/20
Otherwise you get nothing.

Expected Value = P1*V1 + P2*V2 + P3*V3 + P4*V4

.98 or so

$$EV=\frac{1}{2500000}(100{,}000)+\frac{1}{15000}(10000)+\frac{1}{20}(10)+(\ )(\text{zero})$$

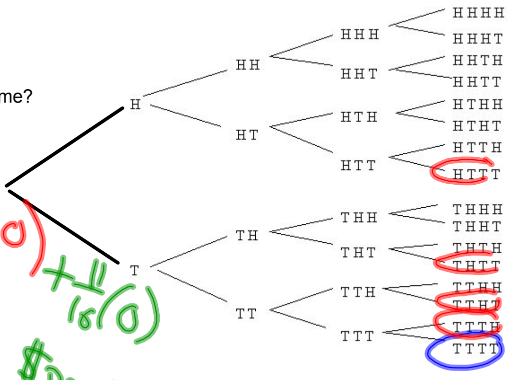
$= 1.566$

Not worth 2$ per ticket

Let's play a different game.

I am running the following game at a county fair.
You pay $5 to play against me.
Suppose we flip a four coins at the same time.
If they are all tails, I pay you $30 dollars.
If there are exactly three tails, I pay you $10 dollars

Do you think it is a good idea to play the game?



$$EV = \frac{1}{16}(30) + \left(\frac{4}{16}\right)(10) + \frac{11}{16}(0)$$

$$= \frac{70}{16} = 4.38 \text{ \$ per game}$$

# of ways to get 1 head in 4 flips

___ ___ ___ ___ $= {}_4C_1$

4 positions

Here is another type of game.

You are a contestant on a game show. You want to win as much money as possible.
The show wants you to win as little money as possible.

You have to pick a ROW in the following grid.

The game show host picks a COLUMN in the following grid.

You win the amount of money shown in the intersection of that ROW and COLUMN

You pick row

				maximin
250	450	280	175	175
425	400	350	380	350 goal
160	260	190	500	160
550	130	325	430	130
550	450	350	500	

I pick this

my goal

Possible Strategies

1. row with highest average
2. pick row with highest possibe payoff
3. pick row with highest worst case payoff

Maximin Strategy

My strategy

Here is another game

You have to pick a ROW in the following grid.

The game show host picks a COLUMN in the following grid.

You win the amount of money shown in the intersection of that ROW and COLUMN

You pick row

Maximin

	17	23	11	worst 11
	13	19	15	13 goal
	21	12	25	12
worst	21 goal	23	25	

I pick column

Minimax

If you know my strategy

you switch to bottom row.

If I realize you are going to switch

I switch too.

repeat.....

If our goals don't match,

there is no stable solution.

Equilibrium Point

Game Theory:

The Game Theory studies winning strategies for parties involved in situations where their interest conflict with each other. Developed by John von Neumann, the theory has applications to real games (cards, chess, etc.), economics, commerce, politics and even military.

Cooperative Games: Or Partial Conflict Games

Suppose you and your friend were arrested last night for a series of bank robberies. The police separated you so that you can't communicate. This morning each of you were given the following options:

Either stay silent or give evidence against your friend

Four outcomes possible:

If both of you stay silent, the police can only link you to one of the robberies. You each would get 6 months in jail

If you blame your friend and he stays silent, you get to go free, while your friend gets 10 years in jail.

If you stay silent and your friend implicates you, he goes free, while you get 10 years in jail.

If you both give evidence against each other, you each get 5 years in jail.

What do you do?

Payoffs (or Outcomes) for Prisoner's Dilema

(10,0)
Your friend'
You

		Your Friend		
		Silent	Rat Out	
You	Silent	(3,3)	(10,0)	10
	Rat Out	(0,10)	(5,5)	5 goal worst case
	wrst	10	5 goal	

http://www.gametheory.net/Mike/applets/PDilemma/PDilemma.html

What is the equilibrium point of this example? Yes, (5,5) is equilibrum point

both side meet goal

No incentive to switch.

What would be best for both of you?

(3,3) silent both

Not equilibrum

There is incentive to switch strategies.

In environmental studies, the Prisoner's Dilema is evident in crises such as global climate change. All countries will benefit from a stable climate, but any single country is often hesitant to curb CO2 emissions.

The immediate benefit to an individual country to maintain current behavior is perceived to be greater than the eventual benefit to all countries if behavior was changed, therefore explaining the current impasse concerning climate change.

In political science, for instance, the Prisoner's Dilema scenario is often used to illustrate the problem of two states engaged in an arms race.

Both will reason that they have two options, either to increase military expenditure or to make an agreement to reduce weapons.

Either state will benefit from military expansion regardless of what the other state does; therefore, the both incline towards military expansion. The paradox is that both states are acting rationally, but producing an apparently irrational result. This could be considered a corollary to deterrence theory.

http://en.wikipedia.org/wiki/Prisoner's_dilemma

Ignore the Blond - A Beautiful Mind

http://www.youtube.com/watch?v=CemLiSI5ox8

A few quotes from the movie:

Adam Smith said "every man for himself"

"Individual ambition serves the common good"

John Nash "Adam Smith needs revision"

As part of what John Nash talked about in the movie and what he proved, there is always a "nash equilibrium", where no player gains an advantage by changing his strategy. A situation where everyone is has a "pretty good" outcome.

You have to pick a ROW in the following grid.

The game show host picks a COLUMN in the following grid.

You win the amount of money shown in the intersection of that ROW and COLUMN

250	450	280	175
425	400	350	380
160	260	190	500
550	130	325	430

Minimax

Maximin

My Strategy: Pick the Column with the Minimum "Maximum Value"

Your Strategy: Pick the Row with Maximum "Minimum Value"

When the two strategies coincide. This is an "Optimal Solution" for both of us. Neither one would benefit by bluffing and picking something else.

You have to pick a ROW in the following grid.

The game show host picks a COLUMN in the following grid.

You win the amount of money shown in the intersection of that ROW and COLUMN

Row Minima

17	23	11
13	19	15
21	12	25

Column Maxima

My Strategy: Pick the Column with the Minimum "Maximum Value"

Minimax

Your Strategy: Pick the Row with Maximum "Minimum Value"

Maximin

When the two strategies coincide. This is an "Optimal Solution" for both of us. Neither one would benefit by bluffing and picking something else.

Another example: Again You want to pick a column and get a MAX value.
I want to pick a row and get a MIN value.

10	7
5	9

Your strategy would be to pick the second column (with a 7 as worst case)

My strategy would be to pick the bottom row (with 9 as worst case). But...

Then you would change your strategy.... an so on...

How does this play out?

Another type of example: Conflict between a pitcher and a batter.

Pitcher has two choices: Fastball or Curveball

Batter has two choices: Expect fastball or expect Curveball

If Batter "guesses right" he has a much better chance of getting a hit.

If the pitcher "fools the batter"

% chance of getting a hit

Wants High

Maximin

		Pitcher Throws		
		Fast	Curve	
Batter expects	Fastball	30%	20%	20 goal
	Curve	10%	50%	10
	worst for pitcher	30 goal	50	

No equilibrium

Have to pick randomly

There is some math to tell you
Throw 40% fast balls &
60% curves or somethg. like that

Mixed strategy.

		Pitcher Throws	
		Fast	Curve
Batter expects	Fastball	30%	20%
	Curve	10%	50%

So what is your thought process as a batter?

The pitcher wants to throw a fastball, so I'll expect that.

But the pitcher knows I'll expect a fastball so he'll probably throw a curve....

After Thanksgiving we will give the math that gives an "optimal solution". It will be something like
Pitcher throws 60% fastballs and 40% curves.
Batter expects 70% fastballs and 30% curves.

We can figure out the expected percentage of the time the batter gets a hit.

It turns out that if both players follow this strategy, then neither one has any incentive to deviate from that stragegy.