Math 116 Euler Circuit Practice Problems:
For In Class Thursday February 2, 2012

Name_____

8. (a) Draw a connected graph with eight vertices such that each vertex has degree 3.
   (b) Draw a disconnected graph with eight vertices such that each vertex has degree 3.
   (c) Draw a graph with eight vertices such that each vertex has degree 1.

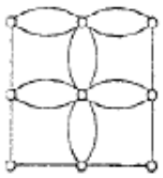
10. Give an example of a connected graph with five vertices, each of degree 4 with
   (a) no loops and no multiple edges.
   (b) loops but no multiple edges.
   (c) multiple edges but no loops.
   (d) both multiple edges and loops.

Exercises 19 and 20 refer to the Green Hills subdivision shown in the map.

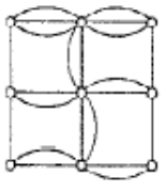
20. A mail carrier must deliver mail on foot along the streets of the Green Hills subdivision. The mail carrier must make two passes on blocks that have houses on both sides of the street (once for each side of the street) and only one pass on blocks that have houses on only one side of the street. Draw a graph that models this situation.

For each graph in Exercises 21 through 26, determine whether the graph has an Euler circuit, an Euler path, or neither of these. Explain your answer. (You do not have to show the actual path or circuit.)

22. (a)

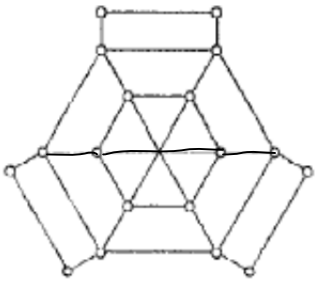


(b)

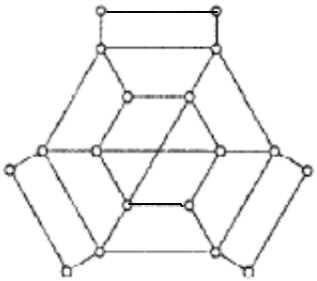


For each graph in Exercises 21 through 26, determine whether the graph has an Euler circuit, an Euler path, or neither of these. Explain your answer. (You do not have to show the actual path or circuit.)

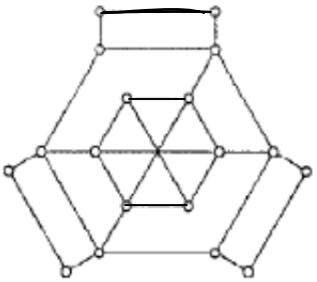
26. (a)



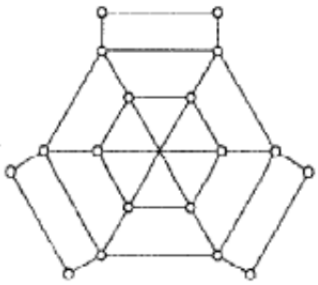
(b)



(c)

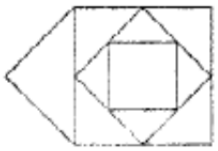


28. Find an Euler circuit for the graph. Show your answer by labeling the edges 1, 2, 3, and so on in the order in which they can be traveled.

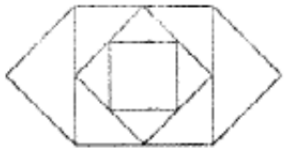


38. (a)

(b)



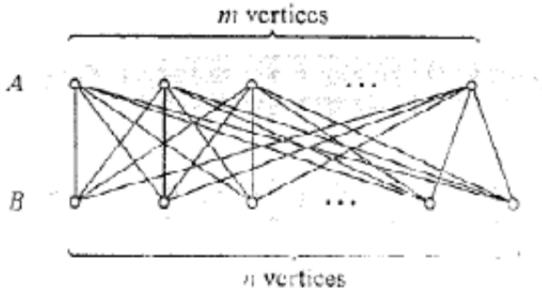
(c)



Jogging Problems (Bonus Points)

54. Suppose a connected graph G has k odd vertices and you want to trace all of its edges. Assuming that you would not trace over any edges twice, what is the least number of times that you would have to lift your pencil? Explain.

56. Complete bipartite graphs. A complete bipartite graph is a graph having the property that the vertices of the graph can be divided into two groups A and B and each vertex in A is adjacent to each vertex in B, as shown in the figure. Two vertices in A are never adjacent, and neither are two vertices in B. Let m and n denote the number of vertices in A and B, respectively.



(a) Explain why if m and n are both even, then the complete bipartite graph has an Euler circuit.

(b) Explain why if $m = 2$ and n is odd, then the complete bipartite graph has an Euler path.