Many Word problems result in Quadratic equations that need to be solved.

Some typical problems involve the following equations:

$$A = L \cdot W \qquad\qquad A = \pi R^2$$

$$a^2 + b^2 = c^2 \qquad\qquad h = -4.9t^2 + v_0 t + h_0$$

Quadratic Equations form Parabolas: $y = ax^2 + bx + c$

Typically there are two types of problems:
1. Find when the equation is equal to zero.
2. Find when the equation has a maximum (or minumum) value.

Methods to find "the roots", or "the zeros", or
"when the graph crosses the x-axis"

    1. Graphing
    2. Factoring
    3. Quadratic Formula

Find the zeros of the following equations by factoring and/or graphing.

$$y = x^2 - 11x + 24 \qquad\qquad y = 3x^2 - 12x - 15$$

$$y = 6x^2 - 7x - 5 \qquad\qquad y = 15x^2 - 11x - 12$$

Word Problems with Quadratic Equations

Quadratic Formula

$$y = ax^2 + bx + c \qquad\qquad 0 = 3x^2 - 4x - 7$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Word Problems with Quadratic Equations

Quadratic Formula

Discriminant

$$b^2 - 4ac$$

$$y = ax^2 + bx + c$$

$$ax^2 + bx + c = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
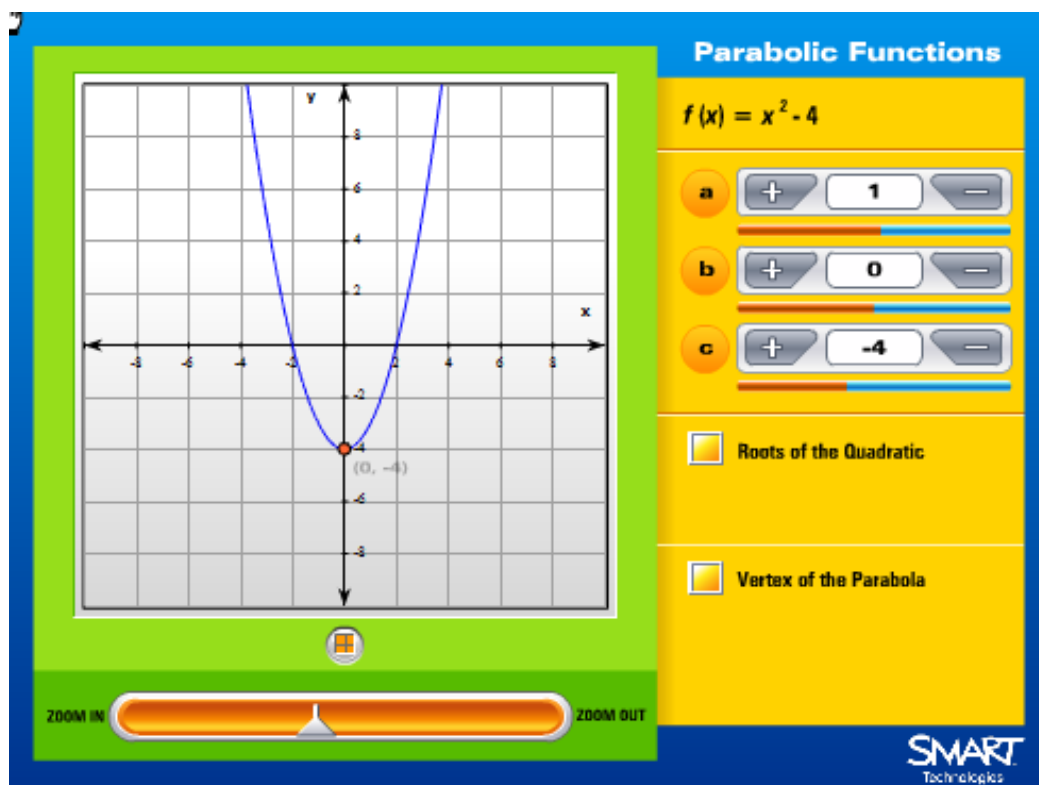
If Discriminant is positive: 2 solutions
If Discriminant is equal to zero : 1 solution
If Discriminant is negative: No solutions

$$0 = x^2 + 2x - 5 \qquad 0 = x^2 + 2x + 1 \qquad 0 = x^2 + 2x + 5$$

Parabolic Functions

$f(x) = x^2 - 4$

a	+	1	−
b	+	0	−
c	+	-4	−

☐ Roots of the Quadratic

☐ Vertex of the Parabola

(0, −4)

ZOOM IN   ZOOM OUT

SMART
Technologies

Maxima and Minima of Quadratic Equations

$$y = ax^2 + bx + c$$
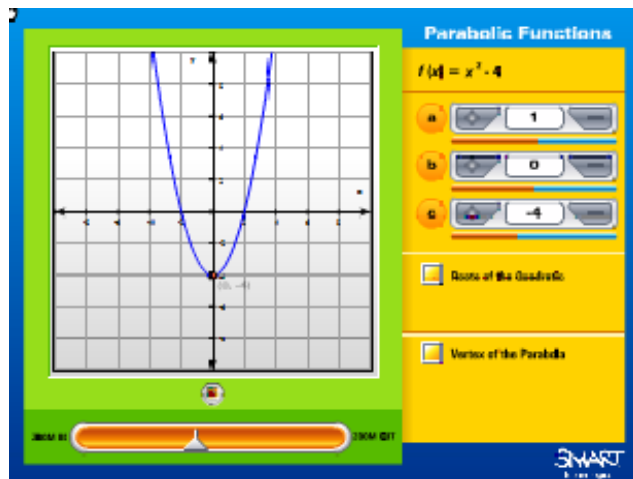
The maximum value of f(x) occurs at $x = \dfrac{-b}{2a}$

If a>0, this is a
<u>minimum</u>
and

If a<0, this is a
<u>maximum</u>

the y-value is

$$f\left(\dfrac{-b}{2a}\right)$$

The vertex also occurs exactly halfway between the two zeros.

Use the factored form to find the zeros of the function.

Use two methods to find the minimum value.

$$y = 3x^2 + 6x - 24$$

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Use the factored form to find the zeros of the function.

Use two methods to find the minimum value.

$$y = 100x^2 - 1500x$$

For many applications with objects falling against gravity, we will use the following equation for the height after 't' seconds.

$$h = -16t^2 + \quad\quad + $$

Initial velocity

Initial height

In the homework, you use "4.9" instead of "16" since the units are "meters per second" rather than "feet per second"

You are standing on a cliff that is 200 feet high. How long will it take for a rock to reach the ground if

- You drop it?

- You throw it downward at an initial speed of 40 feet per second.

Example 1. We are going to fence in a rectangular field and we know that for some reason we want the field to have and enclosed area of 75 sq. ft.
We also know that we want the width of the field to be 3 feet longer than the length of the field.
What are the dimensions of the field?

A corner lot has dimensions of 25 by 40 yards. The city plans to take a strip of uniform width along two sides to widen the streets. How wide should the strip be if the lot is to have an area of 844 square yards?

Example 2.
 Two cars start out at the same point. One car starts out driving north at 25 mph.
Two hours later the second car starts driving east at 20 mph.
How long after the first car starts traveling does it take for the two cars
to be 300 miles apart?

Two trains leave the same city at the same time. The northbound train travels 20 mph faster that the eastbound. If they are 300 miles apart after 5 hours, what is the speed of each train?

Example 3.
 An office has two envelope stuffing machines. Working together they can stuff a batch of envelopes in 2 hours. Working separately it will take the second machine 1 hour longer than the first machine to stuff a batch of envelopes. How long would it take each machine do stuff a batch of envelopes by themselves?

Example:
A motorboat goes 15 miles downstream at top speed, then turns around and goes 15 miles upstream at top speed. The entire trip takes 5 hours. Suppose the top speed in still water is 10 mph. How fast is the current?

Example 4.
A garden measuring 12 meters by 16 meters is to have a pedestrian pathway installed all around it, increasing the total area to 285 square meters. What will be the width of the pathway?

A Bulldozer can move 25 tons of dirt each hour, while an end loader can move only 18 tons of dirt per
hour. The Bulldozer costs $75 per hour to operate, and an end loader costs $50 to operate per hour.
On a certain project, 204 tons of dirt were moved each hour and it cost $600 per hour. How many of each
type of machine were used?

A broker made two investments for a client. The first earned 12% and the second lost 5% of its value. The
total gain of the two accounts was $1040. If the amounts in the two investments had been switched, the result
would have been a gain of $600. Determine how much was actually invested at each rate.

Mixture Problems A 30% salt solution is prepared by mixing a 20% salt solution and a 45% salt solution.
How many liters of each solution must be mixed to obtain 60 liters of the 30% solution?

There are two metal presses used in a machine shop. One is larger than the other. When both are
running, they can produce 88 stamped parts in 8 hours. Another day, the larger press worked 6 hours
while the smaller one worked only 4 hours. In total, they produced 56 stamped parts. Determine the rate
of producing stamped parts for each machine.

Examples: A lumber company owns a 1 mile by 2 mile rectangular forest. They decide to cut two uniformly wide roads through the center of the plot (one north-south and one east-west). How wide should the roads be to result in 1.95 sq. mile of forest remaining?