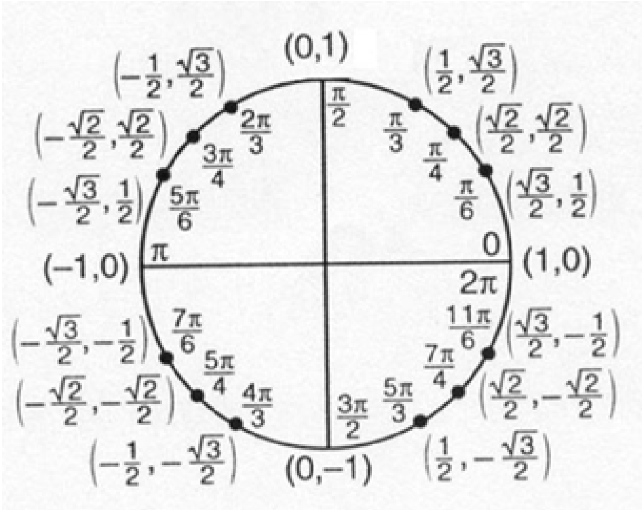
Sec. 6.1 The Unit Circle

(The relationship between angles (measured in radians) and point on a circle of radius 1.)

Sec. 6.2 Trigonometric Functions of Real Numbers

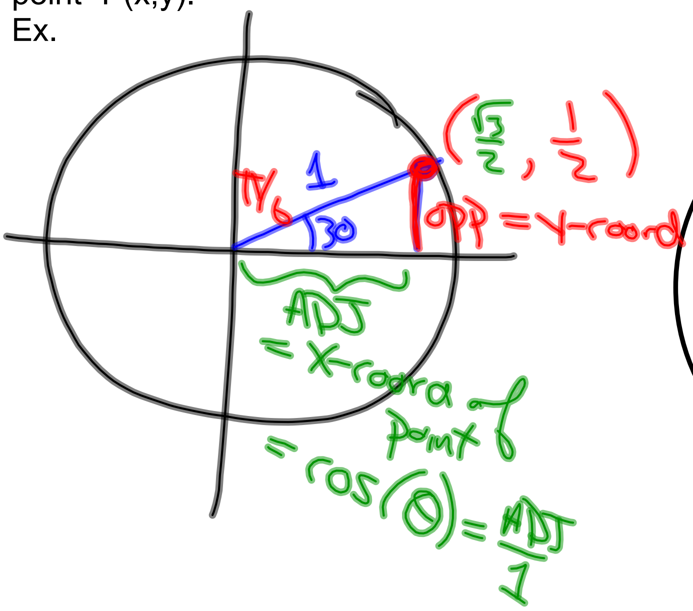
(Defining the trig functions in terms of a number, not an angle.)

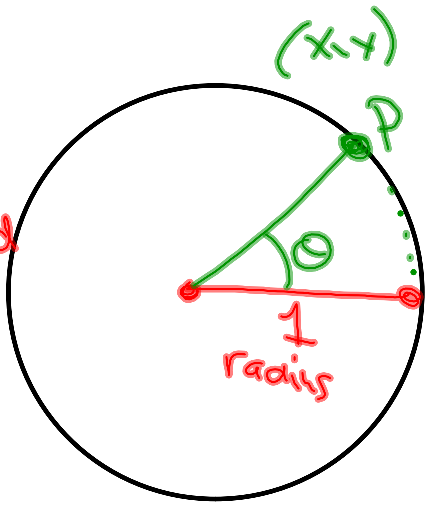


Sec. 6.1 The Unit Circle
Terminal Points on the Unit Circle

Start at the point (1,0) on a unit circle. Walk (counterclockwise) for a distance of ‘t’ units. The point you end up at is called the “terminal point” P(x,y).

Ex.

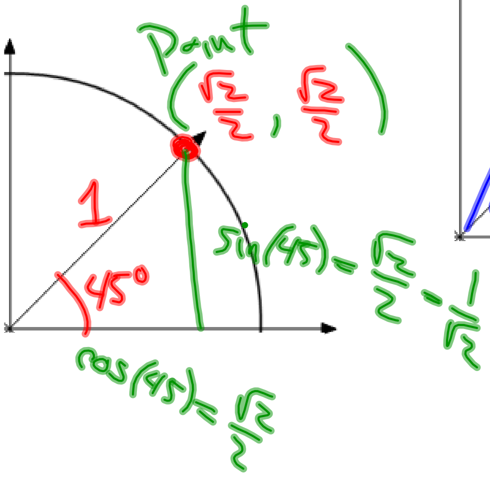


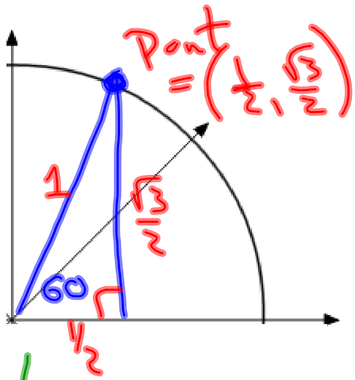


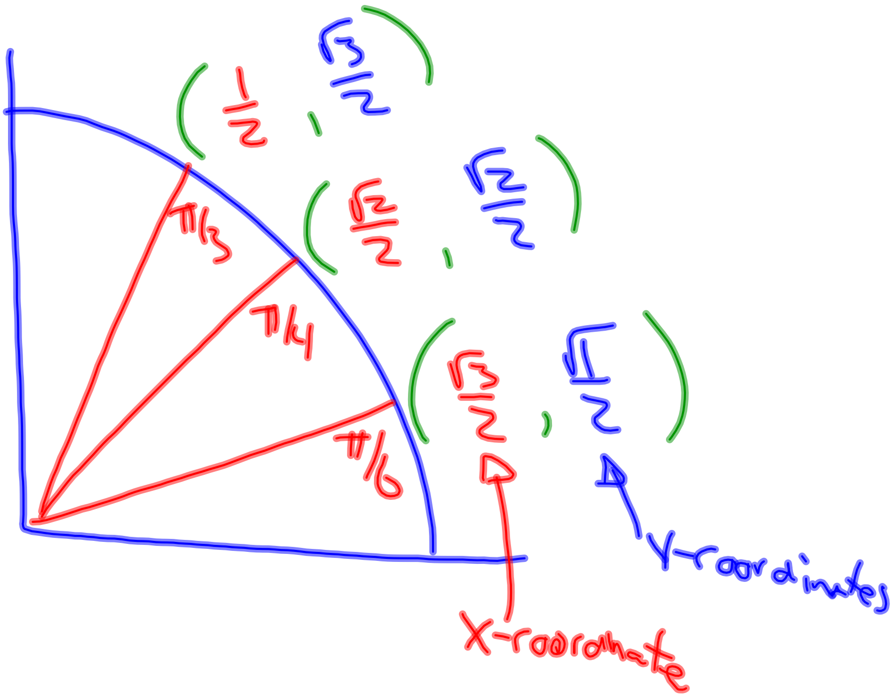
Sec. 6.1 The Unit Circle
Terminal Points on the Unit Circle

Using Right Triangle Trig Functions (from Sec. 5.2) we can find the terminal points for different values of 't'.

Try angles in the first quadrant to begin:







Sec. 6.1 The Unit Circle

Using Reference Numbers to Find Terminal Points

For angles outside the range [0,π/2] we can find the terminal points based on the 'corresponding' terminal point in the first quadrant.

Let t be a real number. The reference number t' associated with t is the shortest distance along the circle between the terminal point 't' and the x-axis.

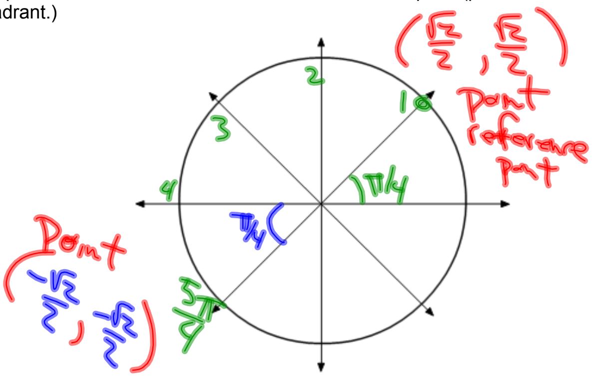
The coordinates of the terminal point come from the coordinates of the reference point (positive or negative depending on the quadrant.)

$t = \frac{5\pi}{4}$

$t' =$

$Q(x, y) =$

$P(x, y) =$

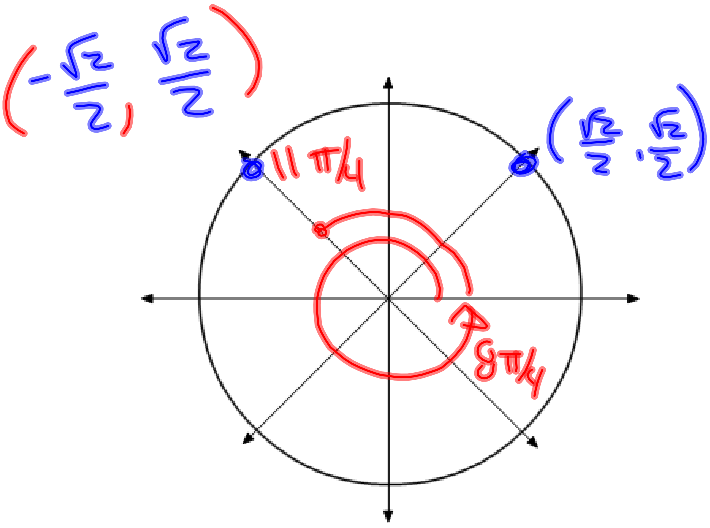


$t = \frac{11\pi}{4}$

$t' =$

$Q(x, y) =$

$P(x, y) =$

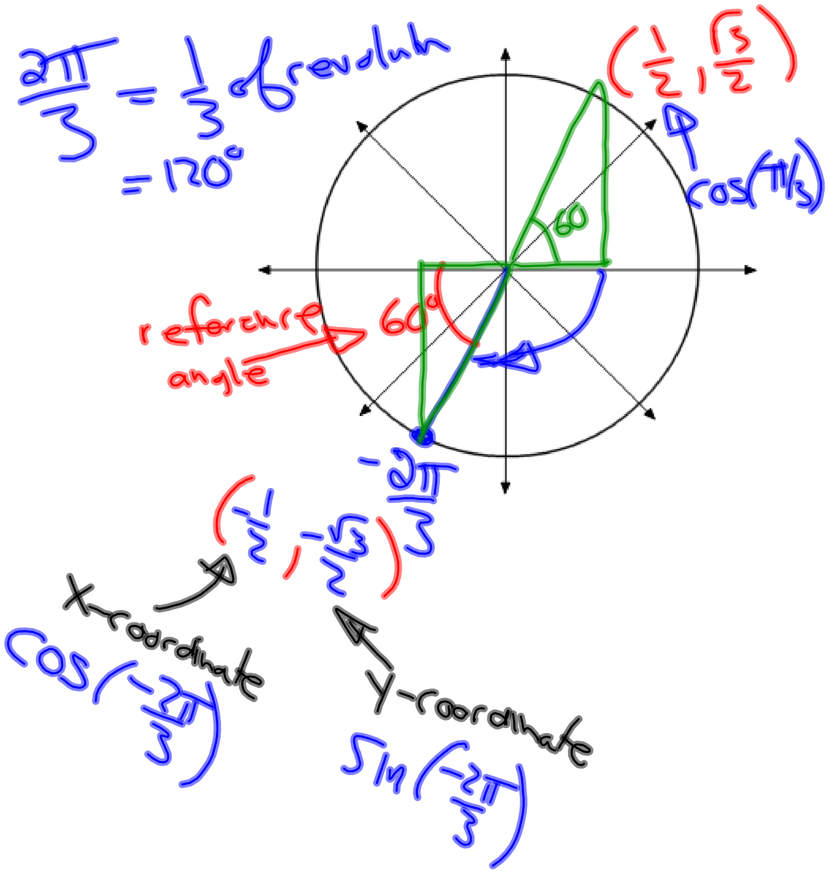


$t = -\frac{2\pi}{3}$

$t' =$

$Q(x, y) =$

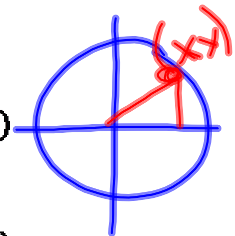
$P(x, y) =$



Sec. 6.2 Trigonometric Functions

Let t be any real number and P(x,y) the terminal point determined by it. Define the trig functions as follows:

$\sin(t) = y$ $\quad\cos(t) = x$ $\quad\tan(t) = \frac{y}{x}\ (x \neq 0)$ $\frac{opp}{ADJ}$



$\csc(t) = \frac{1}{y}\ (y \neq 0)$ $\quad\sec(t) = \frac{1}{x}\ (x \neq 0)$ $\quad\cot(t) = \frac{x}{y}\ (y \neq 0)$

$\csc(t) = \frac{1}{\sin(t)}$ $\quad\sec(t) = \frac{1}{\cos(t)}$ $\quad\cot(t) = \frac{1}{\tan(t)}$

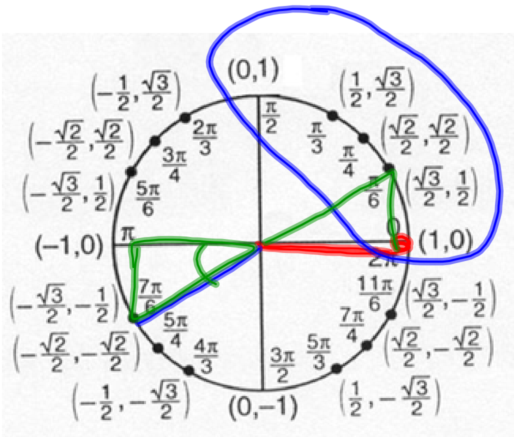
Note that if P(x,y) is in the first quadrant, we can make a right triangle and let 'x' be the length of the adjacent and 'y' be the length of the opposite. The hypotenuse has length '1'. So this is no different that the definitions we had before. But now they are more general.

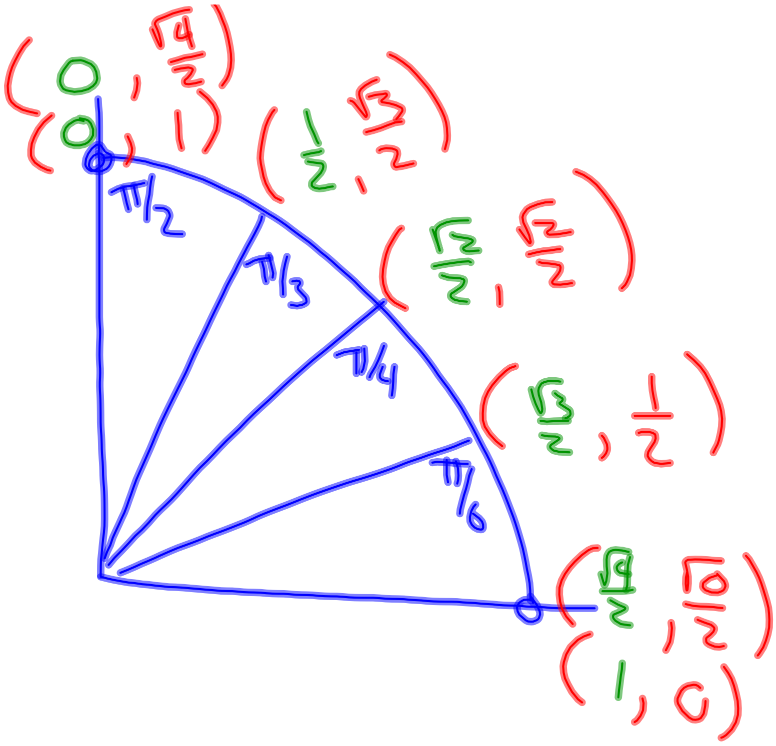
Sec. 6.2 Trigonometric Functions

Special Values

$= y/x = \frac{\text{rise}}{\text{run}} = \text{slope}$

t	$\sin t$	$\cos t$	$\tan t$	$\csc t$	$\sec t$	$\cot t$
0	0	1	0	–	1	–
$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$	2	$2/\sqrt{3}$	$\sqrt{3}/1$
$\pi/4$	$\sqrt{2}/2$	$\sqrt{2}/2$	1	$\sqrt{2}$	$\sqrt{2}$	1
$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}/1$	$2/\sqrt{3}$	2	$1/\sqrt{3}$
$\pi/2$	1	0	–	1	–	0





Sec. 6.2 Trigonometric Functions

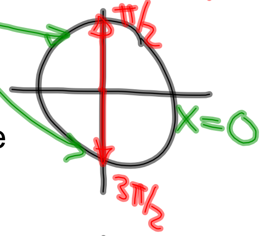
Domains:

sin, cos Domain = all angles all real numbers.

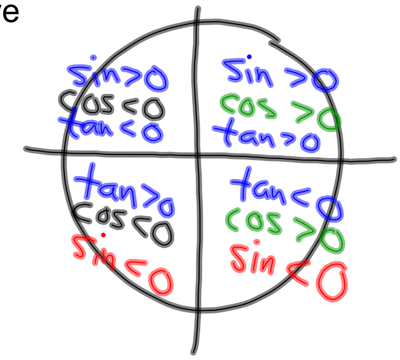
tan, sec (x value in denominator) $\tan(t) = \frac{y}{x}$ $\sec(t) = \frac{1}{x}$

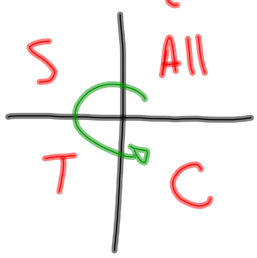
cot, csc (y-value in denominator)

Bad when $y=0$ $t=0$ $t=\pi$



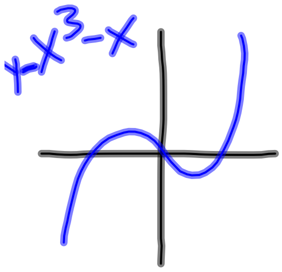
Quadrants where Trig Functions are positive





Sec. 6.2 Trigonometric Functions

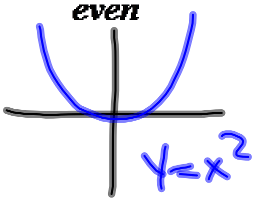
Even and Odd Properties:



$\sin(-t) = -\sin(t)$ — *odd*

$\cos(-t) = \cos(t)$ — *even*

$\tan(-t) = -\tan(t)$ — *odd*



Fundamental Identities

$$\csc(t) = \frac{1}{\sin(t)} \quad \sec(t) = \frac{1}{\cos(t)} \quad \cot(t) = \frac{1}{\tan(t)}$$

$$\tan(t) = \frac{\sin(t)}{\cos(t)} \quad \cot(t) = \frac{\cos(t)}{\sin(t)}$$

reciprocal identities

$$\sin^2(t) + \cos^2(t) = 1 \quad \tan^2(t) + 1 = \sec^2(t) \quad 1 + \cot^2(t) = \csc^2(t)$$

Midterm

5)

$$5^{x-2} = 2^x$$

$$\ln\left(5^{x-2}\right) = \ln\left(2^x\right)$$

$$(x-2)\ln(5) = x \cdot \ln(2)$$

$$x \cdot \ln(5) - 2\ln(5) = x\ln(2)$$

$$x \cdot \ln(5) - x\ln(2) = 2\ln(5)$$

$$x\left(\ln(5) - \ln(2)\right) = 2\ln(5)$$

$$\boxed{x = \frac{2\ln(5)}{\ln(5) - \ln(2)}}$$

5.1 HW

7. A table saw blade, with diameter of 10.00 inches, rotates at about 3450 revolutions per minute. Find the linear speed of a point on the tip of the blade and express it to the nearest mile per hour.
A) 103 mph B) 113 mph C) 123 mph D) 133 mph E) 143 mph

$\pi \cdot$ Diameter

$$\frac{3450\ \cancel{rev}}{1\ min}\cdot\frac{10\pi\ in}{1\ \cancel{rev}}\left(\frac{1\ ft}{12\ in}\right)\left(\frac{1\ mile}{5280\ ft}\right) = \_\_\_\_\_ \frac{miles}{min}$$

2. Evaluate csc 7 to four decimal places.
A) 1.3221 B) 1.4221 C) 1.5221 D) 1.6221 E) 1.7221

radians degrees

$\csc(7) =$ $\csc(7^\circ) =$

cosecant

$= \frac{HYP}{OPP}$

$= \frac{1}{\sin(7)}$

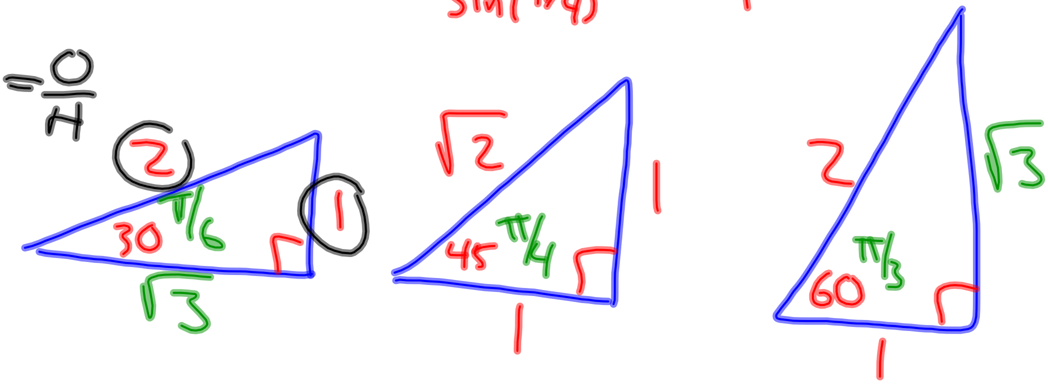
8. Without using a calculator, determine the values for the following 3 quantities: $\sin(\frac{1}{6}\pi)$; $\csc(\frac{1}{4}\pi)$;$\tan(\frac{1}{6}\pi)$.
A) $\frac{1}{2}$; $\sqrt{2}$; $\frac{\sqrt{3}}{3}$ B) $\frac{\sqrt{3}}{2}$; $\sqrt{2}$; $\sqrt{3}$ C) $\frac{\sqrt{3}}{2}$; $\frac{2\sqrt{3}}{3}$; 1 D) $\frac{\sqrt{2}}{2}$; 2; $\frac{\sqrt{3}}{3}$ E) $\frac{\sqrt{2}}{2}$; $\sqrt{2}$; 1

$\sin(\pi/6) = 1/2$

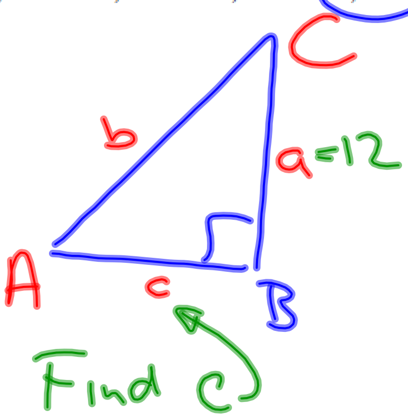
$\csc(\pi/4) = \frac{HYP}{OPP} = \frac{\sqrt{2}}{1}$

$\frac{1}{\sin(\pi/4)}$

$S = \frac{O}{H}$



4. In a triangle with right angle at B, find the length c given that $\tan A = \frac{3}{11}$ and $a = 12$.
A) 35 B) 38 C) 41 D) 44 E) 47



$\text{Tan}(A) = \frac{3}{11} = \frac{OPP}{ADJ} =$

$\frac{3}{11} = \frac{a}{c}$

$\boxed{\frac{3}{11} = \frac{12}{c}}$