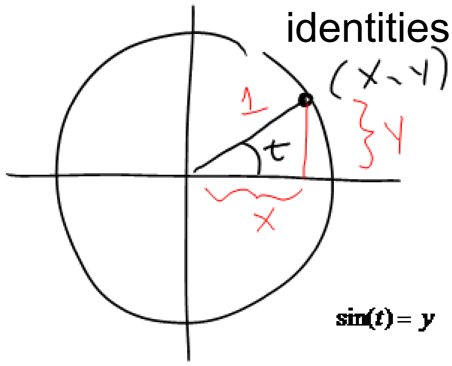
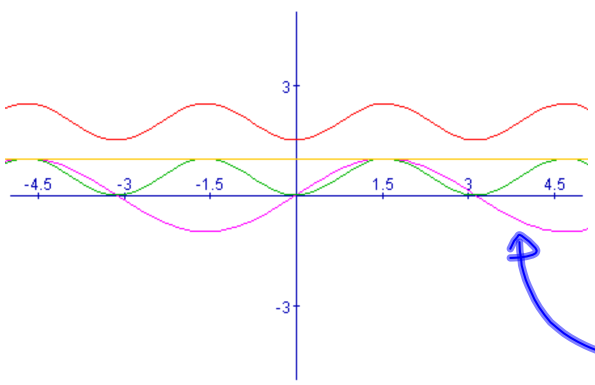
Sec. 7.1 Trigonometric Identities
Sec. 7.2 Addition and Subtraction Formulas

In this chapter, we will be simplifying and factoring expressions that involve trigonometric functions.
To do this we will need to use a number of trigonometric identities and formulas.



$\sin(t) = y \qquad \cos(t) = x \qquad \tan(t) = \frac{y}{x} \quad (x \neq 0)$

$\csc(t) = \frac{1}{y} \quad (y \neq 0) \qquad \sec(t) = \frac{1}{x} \quad (x \neq 0) \qquad \cot(t) = \frac{x}{y} \quad (y \neq 0)$



$2 - \frac{1}{2}\cos(2x)$

$y = \sin^2(x)$

$y = \sin(x)$

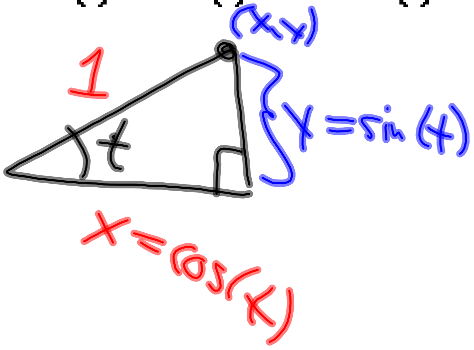
Reciprocal Identities

$$\csc(\theta)=\frac{1}{\sin(\theta)} \qquad \sec(\theta)=\frac{1}{\cos(\theta)} \qquad \cot(\theta)=\frac{1}{\tan(\theta)}$$

$$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)} \qquad \tan(\theta)=\frac{y}{x}=\frac{\sin(\theta)}{\cos(\theta)}$$

Pythagorean Identities

$$\sin^2(t)+\cos^2(t)=1 \qquad \tan^2(t)+1=\sec^2(t) \qquad 1+\cot^2(t)=\csc^2(t)$$



$$\frac{\sin^2(t)}{\cos^2(t)}+\frac{\cos^2(t)}{\cos^2(t)}=\frac{1}{\cos^2(t)}$$

$$\boxed{\tan^2(t)+1=\sec^2(t)}$$

Also $\sin^2(t)+\cos^2(t)=1$

Two more identities

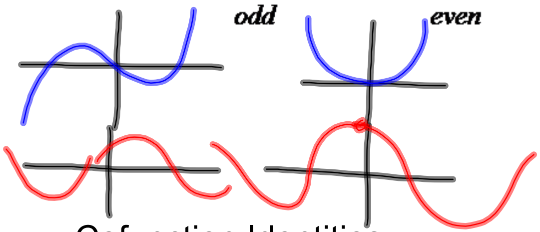
$$\sin^2(t)=1-\cos^2(t)$$

$$\cos^2(t)=1-\sin^2(t)$$

Even - Odd Identities

$$\sin(-t) = -\sin(t) \qquad \cos(-t) = \cos(t) \qquad \tan(-t) = -\tan(t)$$

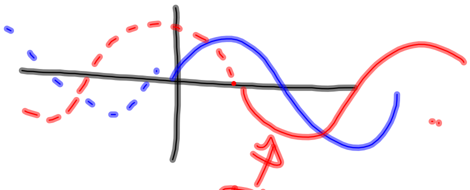
odd *even* *odd*



Cofunction Identities

$$\sin\left(\frac{\pi}{2}-u\right)=\cos(u) \qquad \tan\left(\frac{\pi}{2}-u\right)=\cot(u) \qquad \sec\left(\frac{\pi}{2}-u\right)=\csc(u)$$

$$\cos\left(\frac{\pi}{2}-u\right)=\sin(u) \qquad \cot\left(\frac{\pi}{2}-u\right)=\tan(u) \qquad \csc\left(\frac{\pi}{2}-u\right)=\sec(u)$$



Flip upside down
Shift by $\pi/2$ becomes "cosine"

Guidelines for Proving New Identities

- Start with one side of the equation.
- Use known identities to transform that side to make it equal to the other side.
- Try some of the following ideas if you are stuck:
- Get common denominators
- Change all trig functions to ‘sin’ and ‘cos’.
- Keep in mind the expression that is your goal.
- Be careful not to “do the same thing to both sides of the equation”. With trig functions, it is easier to accidentally turn a false statement into a true one by doing this.

Prove the following:

$$\cos(\theta)(\sec(\theta)-\cos(\theta))=\sin^2(\theta)$$

Start with

$$\cos(\theta)\left(\sec(\theta)-\cos(\theta)\right)$$

$$=\cos\theta\cdot\sec\theta-\cos^2\theta \qquad \text{algebra}$$

$$=\cos\theta\cdot\frac{1}{\cos\theta}-\cos^2\theta \qquad \text{by } \sec\theta=\frac{1}{\cos\theta}$$

$$=1-\cos^2\theta \qquad \text{algebra}$$

$$=\sin^2\theta \qquad \text{by Pythagean Identity}$$

Prove the following:

$$1-\frac{\cos^2(x)}{1+\sin(x)}=\sin(x)$$

$$1-\frac{\cos^2(x)}{1+\sin(x)}$$

difference of squares

$$=1-\frac{1-\sin^2(x)}{1+\sin(x)}=1-\frac{(1-\sin(x))\cancel{(1+\sin(x))}}{\cancel{1+\sin(x)}}$$

$$=1-(1-\sin(x))$$

$$=\sin(x)$$

QED

Prove the following:

$$2\tan(x)\sec(x) = \frac{1}{1-\sin(x)} - \frac{1}{1+\sin(x)}$$

$2\tan(x)\sec(x)$

in terms of cos + sin

$$2\,\frac{\sin(x)}{\cos(x)}\cdot\frac{1}{\cos(x)}$$

$$= \frac{2\sin(x)}{\cos^2(x)}$$

$$= \frac{2\sin(x)}{1-\sin^2(x)} = \frac{1+\sin x - 1 + \sin(x)}{(1-\sin^2(x))}$$

$$= \frac{(1+\sin x) - (1-\sin(x))}{(1-\sin(x))(1+\sin(x))}$$

common denominator

$$= \frac{(1+\sin x)}{(1+\sin x)}\cdot\frac{1}{1-\sin(x)} - \frac{1}{1+\sin(x)}\cdot\frac{(1-\sin x)}{(1-\sin x)}$$

work backwards

Prove the following:

$$\frac{1+\tan(x)}{1+\cot(x)} = \tan(x)$$

$$\frac{1+\tan(x)}{1+\cot(x)} = \left(\frac{\cos}{\cos}\right)\frac{1+\frac{\sin(x)}{\cos(x)}}{1+\frac{\cos(x)}{\sin(x)}}$$

Common denominators

$$= \frac{\left(\frac{\cos(x)+\sin(x)}{\cos(x)}\right)}{\left(\frac{\sin(x)+\cos(x)}{\sin(x)}\right)}$$

$$\cancel{\frac{3+4}{3} = 4}$$

multiply by reciprocal

$$= \left(\frac{\cancel{\cos(x)+\sin(x)}}{\cos(x)}\right)\cdot\left(\frac{\sin(x)}{\cancel{\sin(x)+\cos(x)}}\right)$$

$$= \frac{\sin(x)}{\cos(x)} = \tan(x)$$

☺

Prove the following:

$$\frac{\tan(u)}{\csc(u)} = \sec(u) - \cos(u)$$

$$\frac{\tan(u)}{\csc(u)} = \frac{\left(\frac{\sin(u)}{\cos(u)}\right)}{\left(\frac{1}{\sin(u)}\right)} = \left(\frac{\sin(u)}{\cos(u)}\right)\left(\frac{\sin(u)}{1}\right)$$

$$= \frac{\sin^2(u)}{\cos(u)} = \frac{1-\cos^2(u)}{\cos(u)}$$

$$= \frac{1}{\cos(u)} - \frac{\cos^2(u)}{\cos(u)}$$

$$= \sec(u) - \cos(u)$$

Prove the following:

$$\tan(x) + \cot(x) = \sec(x)\csc(x)$$

Prove the following:

$$\frac{1-\sin(u)}{\cos(u)} = \frac{\cos(u)}{1+\sin(u)}$$

$$(1-\sin(u))(1+\sin(u)) = \cos^2(u)$$

$$1-\sin^2(u) =$$

$$\cos^2(x) = \cos^2(u).$$

Can you do it without multiplying both sides at same time.

$$\frac{1-\sin(u)}{\cos(u)} = \frac{\cos(u)}{1+\sin(u)}$$

$$\frac{1-\sin(u)}{\cos(u)} = \frac{1-\sin(u)}{\cos(u)} \cdot \frac{1+\sin(u)}{1+\sin(u)}$$

$$= \frac{1-\sin^2(u)}{\cos(u)(1+\sin(u))}$$

$$= \frac{\cos^2(u)}{\cancel{\cos(u)}(1+\sin(u))}$$

$$= \frac{\cos(u)}{1+\sin(u)}$$

Prove the following:

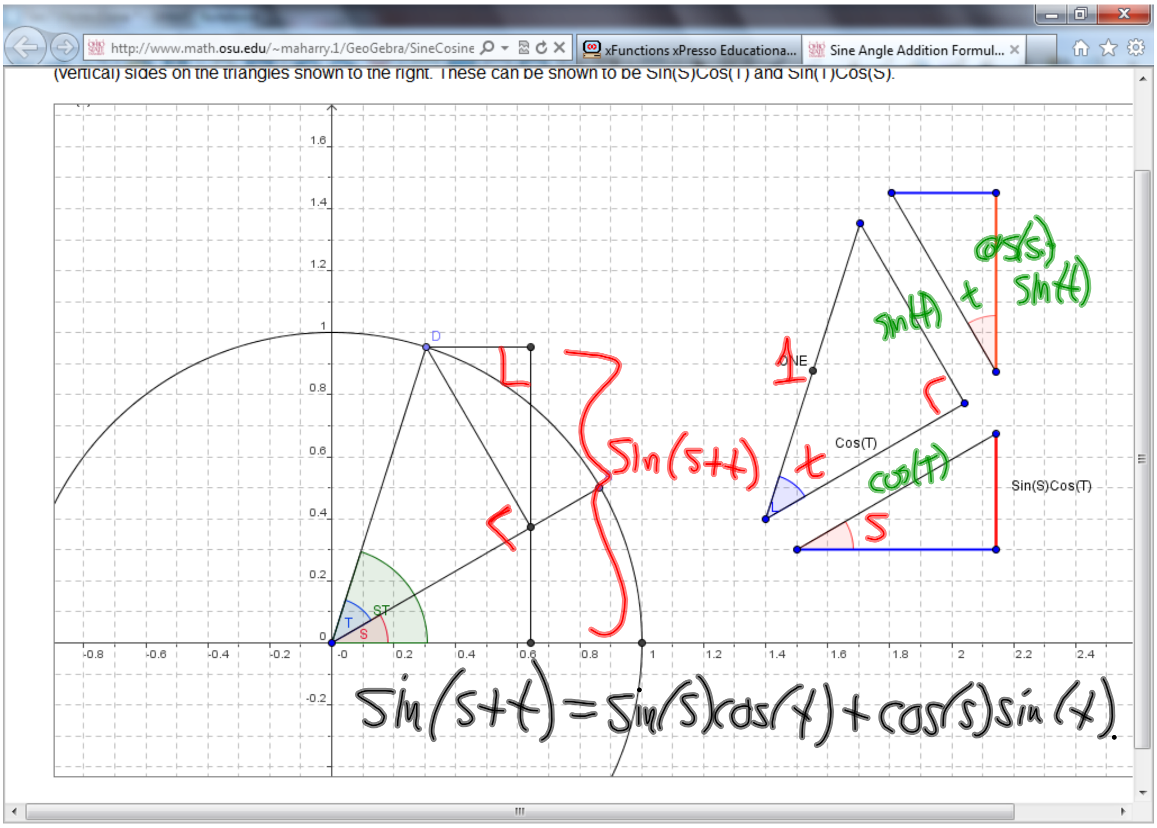
$$\frac{1-\sin(x)}{1+\sin(x)}=(\sec(x)-\tan(x))^2$$

Addition and Subtraction Formulas

$$\sin(s+t)=\sin(s)\cos(t)+\cos(s)\sin(t)$$
$$\sin(s-t)=\sin(s)\cos(t)-\cos(s)\sin(t)$$
$$\cos(s+t)=\cos(s)\cos(t)-\sin(s)\sin(t)$$
$$\cos(s-t)=\cos(s)\cos(t)+\sin(s)\sin(t)$$

$$\tan(s+t)=\frac{\tan(s)+\tan(t)}{1-\tan(s)\tan(t)}$$
$$\tan(s-t)=\frac{\tan(s)-\tan(t)}{1+\tan(s)\tan(t)}$$

http://www.math.ohio-state.edu/~maharry/GeoGebra/SineCosineAdditionFormulas.html



$$\sin(30^\circ+45^\circ)=?$$

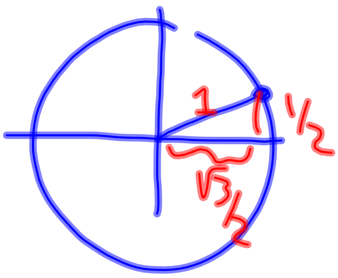
$$=\sin(30)\cos(45)+\cos(30)\sin(45^\circ)$$

$$=\frac{1}{2}\cdot\frac{\sqrt{2}}{2}+\frac{\sqrt{3}}{2}\cdot\frac{\sqrt{2}}{2}$$

$$=\boxed{\frac{\sqrt{2}+\sqrt{6}}{4}}$$

$$\sin(30^\circ)=$$

$$\sin(\pi/6)=\frac{1}{2}$$



$$\sin(45^\circ-30^\circ)=?$$

How to use Addition and Subtraction Formulas

Use the following identity

$$\sin(s-t)=\sin(s)\cos(t)-\cos(s)\sin(t)$$

To show the cofunction identity:

$$\sin\left(\frac{\pi}{2}-u\right)=\cos(u)$$

$$=\sin(\pi/2)\cos(u)-\cos\left(\frac{\pi}{2}\right)\sin(u)$$

$$=1\cdot\cos(u)-0\cdot\sin(u)$$

$$=\cos(u).$$

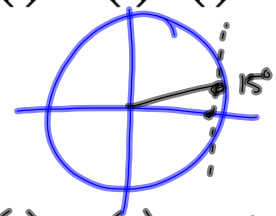
How to use Addition and Subtraction Formulas

Using the values of the trig functions at certain angles, we can determine the values at new angles.

$$\sin(s-t)=\sin(s)\cos(t)-\cos(s)\sin(t)$$
$$\cos(s-t)=\cos(s)\cos(t)+\sin(s)\sin(t)$$

$$\sin\left(\frac{\pi}{12}\right)=\sin\left(\frac{4\pi}{12}-\frac{3\pi}{12}\right)$$

use formula.



$$\cos(15^\circ)=\cos(45^\circ-30^\circ)=\cos(45)\cos(30)+\sin(45)\sin(30)$$

$$=\frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}\frac{1}{2}$$

$$=\boxed{\frac{\sqrt{6}+\sqrt{2}}{4}}=.965$$

$$\cos(165^\circ)=$$

$$\cos(180-15)$$

$$\cos(120+45)$$

$$\cos(135+30)$$

If Sin(x)=1/3, and 'x' is in the 2nd quadrant, draw a rough sketch of the angle and find the exact values of

$\cos(x) =$

$\sin\left(x+\frac{\pi}{6}\right) =$

$\cos\left(x-\frac{\pi}{3}\right)$

Thursday Quiz

(2) Cost per Gigabyte decreases by 9% each year

When will it reduce to 1/2 of todays cost?

$C = P\cdot(1-.09)^{(\ )}$

$\boxed{\frac{1}{2}P = P\cdot(.91)^t}$ Solve for t.

or

$C = P\cdot e^{(-.09)t}$

$\boxed{e^{-.09} = .9139}$

Hw 6.4

3. Determine the period of $y = 5\sin(\frac{4}{9}x + \frac{3\pi}{4})$

A) $\frac{9}{10}\pi$ B) $\frac{9}{4}\pi$ C) 3π D) 18π E) $\frac{9}{2}\pi$

$$\text{Period} = \frac{2\pi}{K} \qquad Y = 5\sin\left(\frac{4}{9}x + \frac{3\pi}{4}\right)$$

$$5\sin\left(\frac{4}{9}(x - ?)\right)$$

$$= 2\pi / (4/9)$$

$$= 2\pi \cdot 9/4 = \frac{18\pi}{4} = \frac{9\pi}{2}$$