Sec 7.2 Addition/Subtraction Formulas
Sec. 7.3 Double-Angle, Half-Angle and Product-Sum Formulas

Previous Trig Identities

$$\csc(\theta)=\frac{1}{\sin(\theta)} \qquad \sec(\theta)=\frac{1}{\cos(\theta)} \qquad \cot(\theta)=\frac{1}{\tan(\theta)}$$

$$\tan(\theta)=\frac{\sin(\theta)}{\cos(\theta)} \qquad \cot(\theta)=\frac{\cos(\theta)}{\sin(\theta)}$$

$$\sin^2(t)+\cos^2(t)=1 \quad \tan^2(t)+1=\sec^2(t) \quad 1+\cot^2(t)=\csc^2(t)$$

$$\sin\left(\frac{\pi}{2}-u\right)=\cos(u) \quad \tan\left(\frac{\pi}{2}-u\right)=\cot(u) \quad \sec\left(\frac{\pi}{2}-u\right)=\csc(u)$$
$$\cos\left(\frac{\pi}{2}-u\right)=\sin(u) \quad \cot\left(\frac{\pi}{2}-u\right)=\tan(u) \quad \csc\left(\frac{\pi}{2}-u\right)=\sec(u)$$

http://www.mnwest.edu/fileadmin/static/website/dmatthews/Geogebra/ShiftIdentity.html

Addition and Subtraction Formulas

$$\sin(s+t)=\sin(s)\cos(t)+\cos(s)\sin(t)$$
$$\sin(s-t)=\sin(s)\cos(t)-\cos(s)\sin(t)$$
$$\cos(s+t)=\cos(s)\cos(t)-\sin(s)\sin(t)$$
$$\cos(s-t)=\cos(s)\cos(t)+\sin(s)\sin(t)$$

$$\tan(s+t)=\frac{\tan(s)+\tan(t)}{1-\tan(s)\tan(t)}$$
$$\tan(s-t)=\frac{\tan(s)-\tan(t)}{1+\tan(s)\tan(t)}$$

For a visual proof of these see these web pages.
The book has an algebraic proof.

http://www.ies.co.jp/math/java/trig/kahote/kahote.html

http://www.math.ohio-state.edu/~maharry/GeoGebra/SineCosineAdditionFormulas.html

Consider the Graph of $\frac{1}{2}Sin(x)+\frac{\sqrt{3}}{2}Cos(x)$

http://math.hws.edu/xFunctions/

Draw a graph of the function with your calulator.

Can we find a nicer equation for it?

Consider the following:

$$ASin(x)+BCos(x)$$

If we find a number ϕ such that $Cos(\phi)=\frac{A}{\sqrt{A^2+B^2}}$ and $Sin(\phi)=\frac{B}{\sqrt{A^2+B^2}}$

So we have the following equation:

$$ASin(x) + BCos(x) = kSin(x + \phi)$$

where $k = \sqrt{A^2 + B^2}$

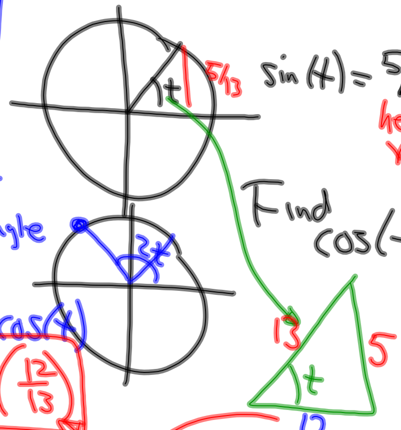
and ϕ such that $Cos(\phi) = \dfrac{A}{\sqrt{A^2 + B^2}}$ and $Sin(\phi) = \dfrac{B}{\sqrt{A^2 + B^2}}$

How could you find a formula for the Sine and Cosine of <u>twice</u> a known angle?

Suppose you know that for a certain angle, Sin(t) = 5/~~12~~13.
How could you use the sum formulas to find the values of the Trig Functions for twice that angle?

$$\sin(2t) = \sin(t + t)$$
$$= \sin(t)\cos(t) + \cos(t)\sin(t)$$
$$= 2\sin(t)\cos(t)$$

if I know a lot about angle t



$\sin(t) = 5/13$
height
y-value

Find $\cos(t)$

y-value at double the angle

$$\sin(2t) = 2\sin(t)\cos(t)$$
$$= \boxed{2\left(\tfrac{5}{13}\right)\left(\tfrac{12}{13}\right)}$$

$$12 = \sqrt{13^2 - 5^2} = \sqrt{144} = 12$$

Double Angle Formulas

$$\sin(2x) = 2\sin(x)\cos(x)$$

$$\cos(2x) = \cos^2(x) - \sin^2(x)$$

$$\boxed{\cos(2x) = 1 - 2\sin^2(x)}$$

$$\cos(2x) = 2\cos^2(x) - 1$$

$$\tan(2x) = \frac{2\tan(x)}{1-\tan^2(x)}$$

If I know information about angle "x" want info about angle "2x"

Proof: Know $\sin^2(x) + \cos^2(x) = 1$

$\cos^2(x) = 1 - \sin^2(x)$

Formulas for Squares of Trig Functions

$$\sin^2(x) = \frac{1-\cos(2x)}{2} = \frac{1}{2} - \frac{1}{2}\cos(2x)$$

$$\cos^2(x) = \frac{1+\cos(2x)}{2}$$

$$\tan^2(x) = \frac{1-\cos(2x)}{1+\cos(2x)}$$

Take previous identities and solve for $\sin^2(x)$ or $\cos^2(x)$

Proof:

Note: What happens when you add the formulas for $\sin^2(x)$ and $\cos^2(x)$?

Half Angle Formulas

$$\boxed{\sin\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1-\cos(u)}{2}}}$$

$u=2x$

$\frac{u}{2}=x$

$$\boxed{\sin^2(x)=\frac{1-\cos(2x)}{2}}=\frac{1}{2}-\frac{1}{2}$$

$$\cos\left(\frac{u}{2}\right)=\pm\sqrt{\frac{1+\cos(u)}{2}}$$

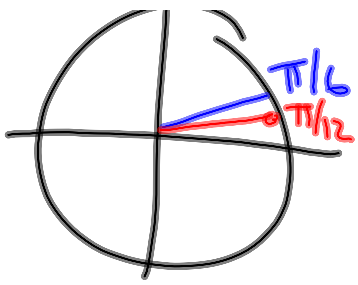
$$\cos^2(x)=\frac{1+\cos(2x)}{2}$$

$$\tan\left(\frac{u}{2}\right)=\frac{1-\cos(u)}{\sin(u)}=\frac{\sin(u)}{1+\cos(u)}$$

$$\tan^2(x)=\frac{1-\cos(2x)}{1+\cos(2x)}$$

Proof: In these, "u" is the known angle we want to find the half-angle "$\frac{u}{2}$"

Applications:



1. Find the exact value of

$$\sin\left(\frac{\pi}{12}\right)=\pm\sqrt{\frac{1-\cos(\pi/6)}{2}}$$

$\pi/12=$ half of $\pi/6$

$$=\pm\sqrt{\frac{1-\sqrt{3}/2}{2}}=0.26$$

Take positive since in QI.

$$\cos\left(\frac{\pi}{12}\right)=\pm\sqrt{\frac{1+\cos\pi/6}{2}}=+\sqrt{\frac{1+\sqrt{3}/2}{2}}=0.97$$

close to 1

Applications:

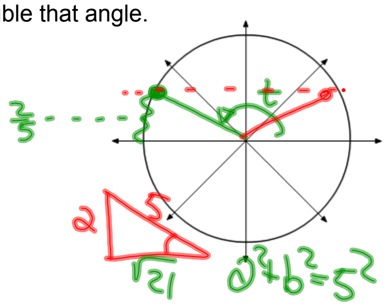
2. Suppose we know that for angle 't', $\sin(t) = \frac{2}{5}$
and 't' is in the 2nd quadrant.
Find the values of 'sin' for double that angle.

$\sin(2t) =$

$2\sin(t)\cos(t)$

$= 2\left(\frac{2}{5}\right)\left(-\frac{\sqrt{21}}{5}\right)$

$= -\frac{4\sqrt{21}}{25}$



Note we don't need to find the actual measure of the angle. We could though use "inverse sine". Do you get the same answer?

$\sin(t) = 2/5$

$t = \sin^{-1}(2/5)$ Sine inverse of 2/5 or arcsin(2/5)

$t = .41$ radians $= 23.57°$

in 2nd Quadrant $t = \pi - .41$ radians $180 - 23.57$

$= 156$

Double angle

$\sin(312°) = -.745$

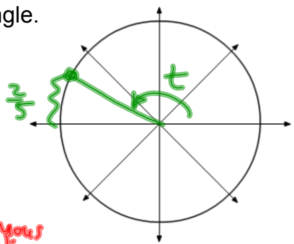
Applications:

2. Suppose we know that for angle 't', $\sin(t) = \frac{2}{5}$
and 't' is in the 2nd quadrant.
Find the value of 'sin' for <u>half</u> that angle.

$\sin\left(\frac{1}{2}t\right) =$

$= \pm\sqrt{\frac{1-\cos(t)}{2}}$

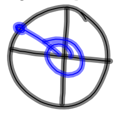
$= +\sqrt{\frac{1-\left(\frac{-\sqrt{21}}{5}\right)}{2}}$ from previous slide



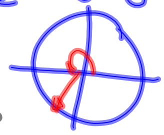
Note we don't need to find the actual measure of the angle. We could though use "inverse sine". Do you get the same answer?

$t = 156°$ or $t = 156 + 360$

so $\frac{t}{2} = 78°$



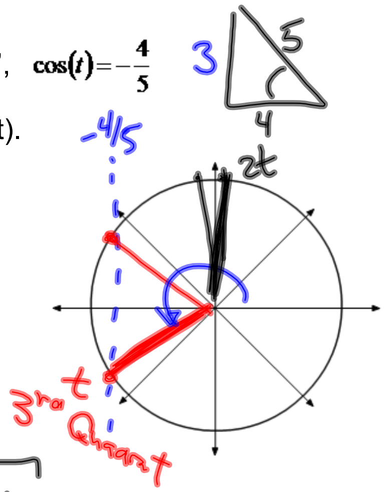
$\frac{t}{2} = 258°$



You pick + or − in square root depending on Quadrant.

Applications:

4. Suppose we know that for angle ‘t’, $\cos(t) = -\frac{4}{5}$
and ‘t’ is in the 3rd quadrant.
Find the values of Sin(2t) and Cos(2t).

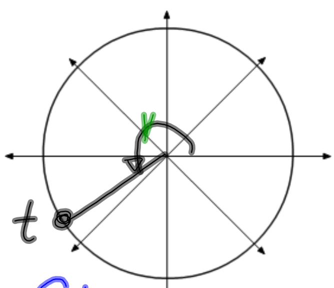


$$\sin(2t) = 2\sin(t)\cos(t) = 2\left(-\frac{3}{5}\right)\left(-\frac{4}{5}\right) = \frac{24}{25}$$

$$\cos(2t) = 2\cos^2(x) - 1 = 2\left(\frac{-4}{5}\right)^2 - 1 = \frac{32}{25} - 1 = \boxed{\frac{7}{25}}$$

Applications:

4. Suppose we know that for angle ‘t’, $\cos(t) = -\frac{4}{5}$
and ‘t’ is in the 3rd quadrant.
Find the values of Sin(t/2) and Cos(t/2).



$$\sin\left(\frac{t}{2}\right) = \pm\sqrt{\frac{1-\cos(t)}{2}} = \pm\sqrt{\frac{1-(-4/5)}{2}} = \sqrt{\frac{9/5}{2}} = \pm .81 \text{ in Q1 or Q2}$$

$t > 180°$
so
$\frac{t}{2} > 90°$
So in QII.

$$\cos\left(\frac{t}{2}\right) = \pm\sqrt{\frac{1+(-4/5)}{2}} = \pm\sqrt{\frac{1/5}{2}} = \pm .01 = \boxed{-.01}$$

Make Up your own Trig Formulas:

What would be a "Triple Angle Formula"? Does it work?

$$\sin(3t) = 3\sin(t)\cos(t)$$ guess

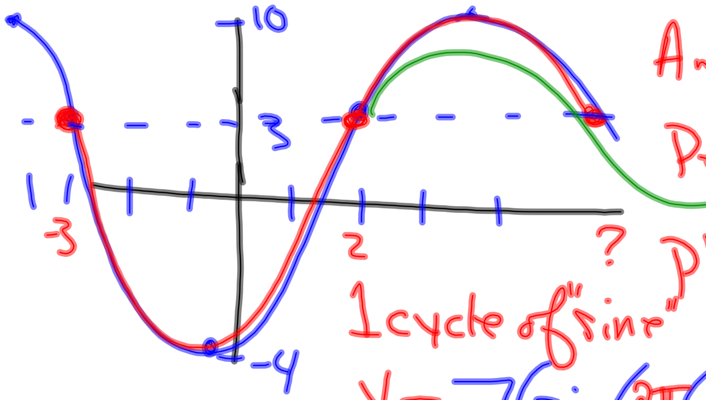
$$\sin(3t) = \sin(2t + t)$$

$$= \sin(2t)\cos(t) + \cos(2t)\sin(t)$$

$$= 2\sin(t)\cos(t)\cdot\cos(t) + (\cos^2(t) - \sin^2(t))\sin(t)$$

Verify the identity $1 - \cos 2x = \tan x \sin 2x$.

Quiz from Friday



Amp 7

Period = 10

k =

Phase shift

1 cycle of "sine"

$$Y = 7\left(\sin\left(\underbrace{\frac{2\pi}{10}}_{k}(x-2)\right)\right)+3$$

or

$$Y = -7\sin\left(\frac{2\pi}{10}(x+3)\right)+3$$

$$\boxed{\text{Period} = \frac{2\pi}{k} \quad \text{and} \quad k = \frac{2\pi}{\text{period}}}$$

#2 population increases by 20% every 3 years

How long to go from 20000 to 40000?

$$40000 = 20000(1.2)^{(t/3)}$$

solve for t.

$$P = 20000\,e^{rt}$$

first find r

then find t when $P = 40000$.

$$20000(1.20) = 20000\,e^{r\cdot 3}$$

$$\ln(1.20) = r\cdot 3$$

$$r = \frac{\ln(1.20)}{3} \cong \frac{.20}{3}$$

$$20000\,e^{.20(t/3)}$$