MSLC – Math 152
Final Exam Review

1. Given the following expansion

$$\frac{1}{\left(1+\frac{8}{n}\right)^{2/3}}*\frac{8}{n}+\frac{1}{\left(1+\frac{16}{n}\right)^{2/3}}*\frac{8}{n}+\frac{1}{\left(1+\frac{24}{n}\right)^{2/3}}*\frac{8}{n}+\ldots+\frac{1}{\left(1+\frac{8n}{n}\right)^{2/3}}*\frac{8}{n}$$

   a. Write the above sum in the form $\sum_{k=?}^{?} a_k$

   b. The limit of this sum is a definite integral: $\lim_{n\to\infty}\sum_{k=1}^{n} a_k = \int_1^b f(x)dx$. Find $f(x)$ & b.

   c. Calculate the limit of the sum.

   d. Suppose the integral is instead $\int_0^b g(x)dx$. Write the new integral. Is the value the same or do you get a different value than before?

2. Given $f(x)=2x^2+x-1$, use a right Riemann Sum to calculate the definite integral $\int_2^5 f(x)dx$.

3. Find a formula for the distance traveled in 5 seconds by an object accelerating at a constant rate of $A\ ^{m}/_{sec^2}$ if the object has a starting velocity of $5\ ^{m}/_{sec}$.

4. Suppose a function is defined as $f(t)=\begin{cases}-t^2+4t & 0\le t<2\\ t^3-6t^2+12t-4 & 2\le t\end{cases}$.

   a. Compute the accumulation (area) function $F(x)=\int_0^x f(t)dt$

      (Hint: since *f(x)* is a piecewise defined function, your answer is likely to be as well.)

   b. Calculate the area contained between x=1 and x=4

   c. Calculate the rate of change of the area at x=3

5. Calculate the following:

   a) $h'(x)$ *if* $h(x)=\int_0^x \sqrt{1+t^2}\,dt$

   b) $F'(x)$ **if** $F(x)=\int_1^{x^3}\frac{1+t}{1+t^2}dt$

   c) $g'(x)$ **if** $g(x)=\int_{x^2}^{x^3}\sin\left(\sqrt{t}\right)dt \quad \left(1\le x^2\le x^3\right)$

   d) $\cancel{\lim_{x\to 0}\frac{\frac{d}{dx}\int_0^x \frac{e^t-1}{e^t+1}dt}{x}}$

6. For what values of p does the integral $\int_1^\infty \frac{1}{x^p}dx$ converge?

7. Suppose a rabbit warren (population) is modeled such that the warren exhibits a growth rate proportional to the square root of its size. If the warren's initial size is 40 rabbits and its size is 250 rabbits after 40 days, find the number of rabbits in the warren after 80 days.

8. A certain cell culture exhibits a growth rate that is proportional to its size. The culture increased by 10-fold in 24 hours and had 1,200 cells at time t=2 hours.
   a. What is the growth rate constant?
   b. What is the growth rate at any time t?
   c. What was the initial size of the culture?
   d. How long did it take for the culture to initially double?

9. A farmer needs to irrigate his field. He has water stored in a tank that has the shape of: $y = x^2$, bounded by x=0, y=0, y=25 and rotated about the y-axis. The water must be pumped out through the top of the tank and into the field. How much work is done pumping the water out of the tank for use in the field?
   (Note – water has a density of 1 gram per cubic centimeter, and all measurements are in the SI system – meters, kilograms and seconds).

10. Find the volumes of the following solids:
    a. The solid bounded by the x-axis, $y = \sqrt{x^2 - 1}$ and x=10 with semicircular cross-sections $\perp$ to the x-axis.
    b. The solid bounded by the functions $y = (x-1)^2$, $y = (x-5)^2$ and y=0 and:
       i. rotated about the line x=10 (use either method to do this)
       ii. rotated about y=4 (again either method)

11. Find the total area enclosed between the curves $x = \sin y$ and $y = \frac{\pi}{2} x$. Note that the graphs intersect when $y = \pm\frac{\pi}{2}$ and when $y = 0$.

12. Find the length of the curve $x = \frac{1}{3}\sqrt{y}(y-3),\ 1 \le y \le 9$.

13. Find the following integrals:

a) $\int_0^2 x^2 \sin(3x)dx$ b) $\int x^3 \sin\left(x^2\right)dx$ c) $\int_{-\infty}^{\infty} xe^{-x^2}dx$

d) $\int_0^2 e^x \sin x dx$ e) $\int_3^{\infty} \frac{2x+1}{(x-2)(x-1)^2}dx$ f) $\int_1^8 \frac{4}{x^{4/3} + 2x - 3x^{2/3}}dx$

g) $\int_0^{1/2} \frac{3}{\sqrt{4-16x^2}}dx$ h) $\int_0^2 \frac{x+1}{x^2+4}dx$ i) $\int_0^1 \arcsin x dx$

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14. Find a number *n* such that $y=\left(x^n+c\right)^{3/2}$ is a solution to the differential equation $2y'-9x^2y^{1/3}=0$. Find the particular solution when $y(2)=1$.

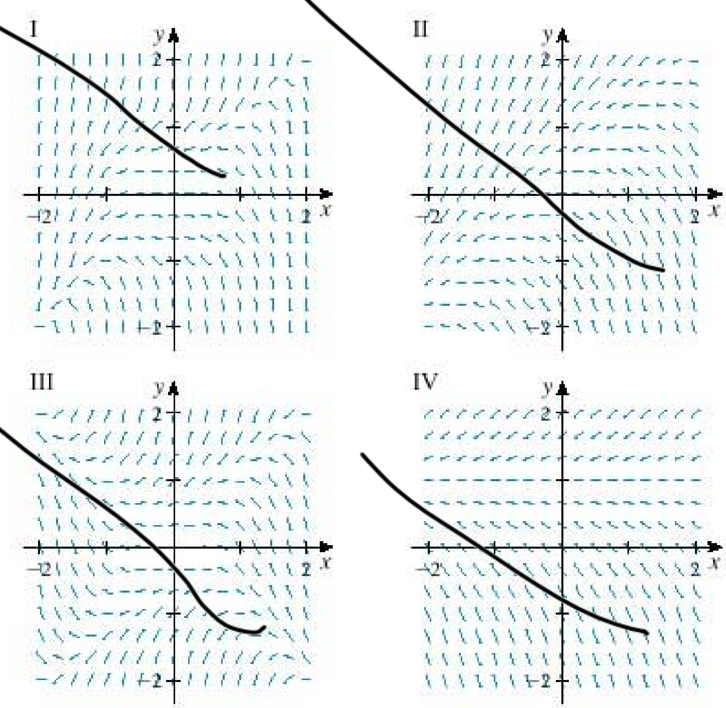
15. Solve the following initial value problems:

a) $\dfrac{dy}{dx}=\dfrac{2x+\sec^2 x}{2y},\ y(0)=-5$

b) $xyy'=\ln x,\ y(1)=2$

16. On a tropical island, there is a colony of wild parakeets which has formed from escaped/released pets which had been imported and purchased by the islands residents. In 2009, there were 400 wild parakeets on this island, and by 2010 the number of wild parakeets had tripled. It is estimated that the maximum number of parakeets that the island can support is 10,000. Assuming that the size of the wild parakeet population follows the logistic model, how long will it take for the population to increase to half of the carrying capacity?

17. Select a direction field for the differential equation $y'=y^2-x^2$ from a set of direction fields labeled I-IV.



18. Use Euler's method with step size 0.25 to estimate $y(1)$, where $y(x)$ is the solution of the initial-value problem below. Round your answer to four decimal places.

$$y'=5x+y^2,\quad y(0)=0$$