7.4 #4

$$\boxed{\int \frac{x^2-5x+16}{(2x+1)(x-2)^2}\,dx} = \frac{A}{2x+1}(x-2)^2 + \frac{B}{x-2}(x-2)(2x+1) + \frac{C}{(x-2)^2}(2x+1)$$

$A+2B=1$

$-4A-3B+2C=-5$

$4A-2B+C=16$

$C \cdot u^{-2}$

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In[2]:= f[x_] = (x^2 - 5 x + 16) / ((2 x + 1) (x - 2)^2)
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Out[2]= $\frac{16-5x+x^2}{(-2+x)^2\,(1+2x)}$

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In[4]:= Apart[f[x]]
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Out[4]= $\frac{2}{(-2+x)^2} - \frac{1}{-2+x} + \frac{3}{1+2x}$

$A=3 \quad C=2$

$B=-1$

$$\text{Integral} = \int \frac{3}{2x+1} - \frac{1}{x-2} + \frac{2}{(x-2)^2}\,dx$$

<u>other</u> typical partial Fracs integrals $\rightarrow$ $\int \frac{3x}{x^2+4}$ $\quad \int \frac{-2}{x^2+4}$

$u = x^2+4$ $\quad \frac{1}{2}\tan^{-1}\left(\frac{x}{2}\right)$

Midterm III Review Problems

From Sec 7.5 List of Integrals to Solve

(3) $$\int \frac{\sin(x) + \sec(x)}{\tan(x)}\,dx$$

$$\int \frac{\sin}{\tan} + \frac{1/\cos}{\sin/\cos}$$

$$\int \cos(x) + \frac{1}{\sin(x)}$$

$$\sin(x) - \ln\left|\csc(x) + \cot(x)\right| + C$$

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$$\int \sec(x) = \ln\left|\sec(x) + \tan(x)\right| + C$$

$$\int \csc(x) = -\ln\left|\csc(x) + \cot(x)\right|$$

(5) $$\int_0^2 \frac{2t}{(t-3)^2}\,dt$$

$u = t - 3$

$du = dt$

$u + 3$

$$\int \frac{2t}{u^2}\,du$$

$$\int \frac{2(u+3)}{u^2}\,du = \int \frac{2u}{u^2} + \frac{6}{u^2}\,du$$

9. $$\int_1^3 r^4 \cdot \ln(r)\, dr$$

$u = \ln(r)$ $dv = r^4 \cdot dr$

$du = \frac{1}{r} dr$ $V = \frac{r^5}{5}$

$$= \ln(r) \cdot \frac{r^5}{5}\Big]_1^3 - \int_1^3 \frac{r^5}{5} \cdot \frac{1}{r} \cdot dr$$

10. $$\int \frac{x-1}{x^2-4x-5}\, dx$$

$$\frac{x-1}{(x-5)(x+1)} = \frac{A(x+1)}{x-5\,(x+1)} + \frac{B\,(x-5)}{x+1\,(x-5)}$$

$$x - 1 = Ax + A + Bx - 5B$$

$$\boxed{\begin{aligned} 1 &= A + B \\ -1 &= A - 5B \end{aligned}}$$

$$\int \frac{A}{x-5} + \frac{B}{x+1}\, dx$$

(11) $\int \frac{x-1}{x^2-4x+5}\,dx$ ← doesnt factor.

$$\int \frac{x-1}{x^2-4x+4+1} = \int \frac{x-1}{(x-2)^2+1}$$

$u = x-2$ $\quad x = u+2$

$du = dx$

$$\int \frac{(u+2)-1}{u^2+1}\,du$$

$$= \int \frac{u}{u^2+1}\,du + \int \frac{1}{u^2+1}\,du$$

$w = u^2+1$ $\quad = \tan^{-1}(u)$

$dw = 2u\,du$

(13) $\int \sin^3(\theta)\cos^5(\theta)\,d\theta$

$$\underbrace{\cos^4(\theta)}\ \underbrace{\cos\theta\, d\theta}_{du}$$

$$u^3(1-u^2)^2$$

$u = \sin\theta$

$du = \cos\theta\, d\theta$

$$\int u^3(1-u^2)^2\,du$$

(17) $\int x \cdot \sin^2(x)\, dx$

$u = x \qquad dv = \sin^2(x)\, dx$

$du = dx \qquad V = ?$ hard

$\int x\left(\frac{1}{2}(1 - \sin 2x)\right) dx$

$\int \frac{1}{2}x - \frac{1}{2}\int x \sin(2x)\, dx$

$u = x \qquad dV = \sin 2x$

$du = dx \qquad V = -\frac{1}{2}\cos(2x)dx$

$\frac{1}{4}x^2 - \left(\frac{1}{2}\left(-\frac{1}{2}\cos(2x)\cdot x - \int -\frac{1}{2}\cos(2x)\,dx\right)\right)$

$\frac{1}{4}x^2 + \frac{1}{4}x\cos(2x) - \frac{1}{4}\frac{1}{2}\sin(2x) + C$

(23) $\int (1 + \sqrt{x})^8\, dx$

$u = 1 + \sqrt{x}$

$\boxed{dx = 2\sqrt{x}\, du}$ $\quad du = +\frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}\, dx$

$\int u^8 \cdot 2\sqrt{x} \cdot du$

$\int u^8 \cdot 2(u - 1)\, du$

27 $\int \frac{e^x \, 1}{e^x(1+e^x)} dx$ $u = 1+e^x$ $du = e^x dx$

$1 = \frac{1}{1+e^x} + \frac{e^x}{1+e^x}$ $\int \frac{e^x}{1+e^x} dx$

$\int \frac{1}{1+e^x} dx$ $\int \frac{1}{u} du$

$= \int 1 - \frac{e^x}{1+e^x} dx$

43 $\int e^x \sqrt{1+e^x}\, dx$

$u = 1+e^x$

$du = e^x dx$

$\int \sqrt{u} \cdot du = \frac{2}{3} u^{3/2} + C$

$\frac{2}{3}(1+e^x)^{3/2} + C$

(49) $$\int \frac{1}{x\sqrt{4x+1}}\,dx$$

$u = 4x+1$

$du = 4\,dx$

$$\frac{1}{4}\int \frac{1}{x\sqrt{u}}\,du$$

$$\frac{1}{4}\int \frac{1}{\left(\frac{u-1}{4}\right)\sqrt{u}}\,du$$

$$\frac{1}{u^{3/2}} - \frac{1}{\sqrt{u}}$$

$$\int \frac{1}{u^{3/2}-\sqrt{u}}\,du$$

(49) $$\int \frac{1}{x\sqrt{4x+1}}\,dx$$

$u = \sqrt{4x+1}$ $\quad u^2-1 = 4x$

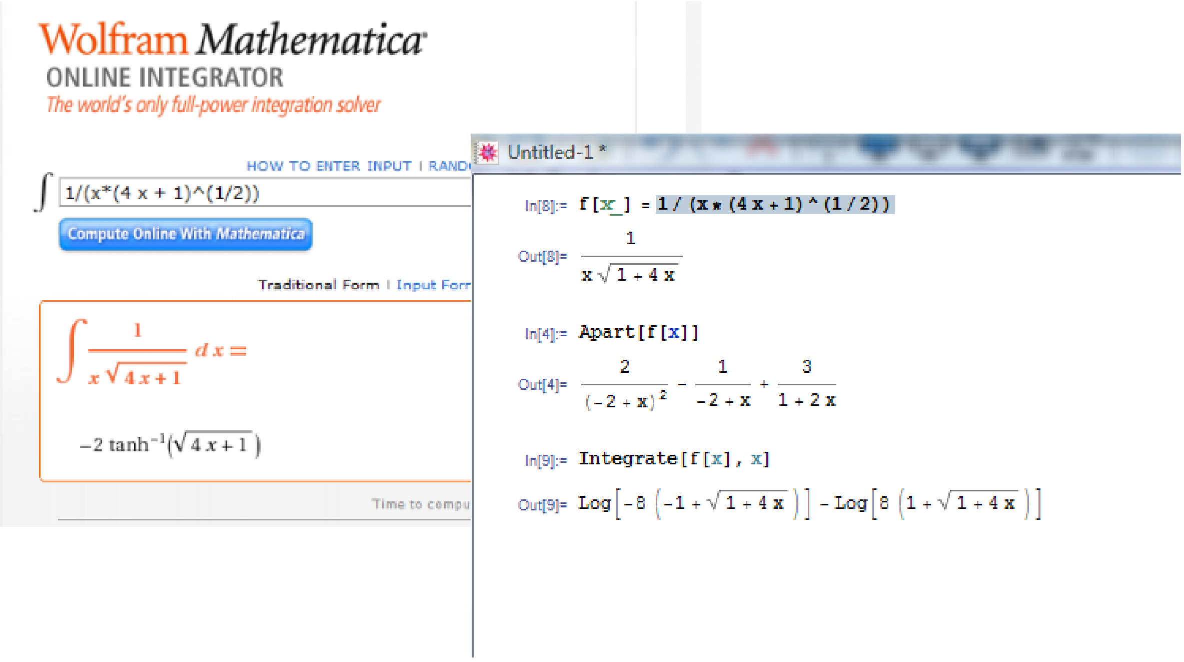
$du = \frac{1}{2\sqrt{4x+1}}\cdot 4\,dx$

$$\frac{1}{2}\int \frac{u}{\left(\frac{u^2-1}{4}\right)\cdot u}\,du$$

$$\int \frac{2}{(u-1)(u+1)}\,du$$

$$= 2A\ln|u-1| + 2B\ln|u+1|$$

$$= 2A\ln\left|\sqrt{4x+1}-1\right| + 2B\ln\left|\sqrt{4x+1}+1\right|$$



(51) $\displaystyle\int \frac{1}{x\sqrt{4x^2+1}}\,dx$

$\boxed{\begin{array}{c}\sqrt{a^2-x^2}\\ x=a\sin\theta\end{array}}$

$\boxed{\begin{array}{c}\sqrt{x^2-a^2}\\ x=a\sec\theta\end{array}}$

$\boxed{\begin{array}{c}\sqrt{x^2+a^2}\\ x=a\tan\theta\end{array}}$

$\sqrt{4x^2+1} = \sqrt{4\left(x^2+\tfrac{1}{4}\right)}$

$= 2\sqrt{x^2+\tfrac{1}{4}}$

$\tfrac{1}{4}=a^2$

$x = \tfrac{1}{2}\tan\theta$

$dx = \tfrac{1}{2}\sec^2\theta$

$\sqrt{4x^2+1} = 2\sqrt{x^2+\tfrac{1}{4}} = \sec\theta$

$\displaystyle\int \frac{1}{\left(\tfrac{1}{2}\tan\theta\right)\cdot\sec\theta}\cdot\tfrac{1}{2}\sec^2\theta\cdot d\theta$

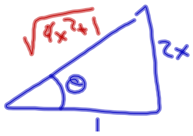
$\displaystyle = \int \frac{\sec\theta}{\tan\theta}\,d\theta$

$\displaystyle = \int \frac{1}{\sin\theta}\,d\theta$

$x = \tfrac{1}{2}\tan\theta$

$\theta = \tan^{-1}(2x)$

$= -\ln\left|\csc\theta + \cot\theta\right| + C$



$= -\ln\left|\frac{\sqrt{4x^2+1}}{2x} + \frac{1}{2x}\right| + C$

7.2 (3) $\int \cos^2(x) \underbrace{\sin(2x)} dx$

$\int \cos^2(x) \cdot 2\sin(x)\cos(x)\, dx$

seems easier

or $\int \frac{1}{2}(1+\sin(2x)) \cdot \sin(2x)\, dx$

ok, but harder.

(10) $\int \frac{1}{x^5\sqrt{9x^2-1}} dx$

$\sqrt{9(x^2-1/9)}$

$3\sqrt{x^2-1/9}$

$\sqrt{x^2-a^2}$

$x = a\sec\theta$

$x = 1/3 \sec\theta$

$dx = \frac{1}{3}\sec\theta\tan\theta\, d\theta$

$\sqrt{x^2-1/9} = \sqrt{\frac{1}{9}\sec^2\theta - 1/9}$

$\frac{1}{3}\cdot\tan\theta$

$\int \frac{1}{\left(\frac{1}{3}\sec(\theta)\right)^5 3\left(\frac{1}{3}\tan\theta\right)} \frac{1}{3}\sec\theta\tan\theta\, d\theta$

$3^4 \int \frac{1}{\sec^4(\theta)} d\theta$

$3^4 \int \cos^4(\theta) d\theta$

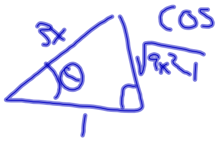
$x = 1/3 \sec\theta$

$\theta = \sec^{-1}(3x)$

$\sec\theta = \frac{3x}{1}$

$\cos(\theta)$

$\cos(\sec^{-1}(3x))$



$= 1/3x$

7. $$\int_0^{2\sqrt{3}} \frac{x^3}{\sqrt{16-x^2}}\,dx$$

$\sqrt{a^2-x^2}$

$$\boxed{\begin{gathered} x = a\sin\theta \\ x = 4\sin\theta \\ dx = 4\cos\theta\, d\theta \\ \sqrt{16-x^2} = 4\cos\theta \end{gathered}}$$

$\theta = \sin^{-1}(x/4)$

$$\int \frac{(4\sin\theta)^3}{4\cos\theta}\, 4\cos\theta\, d\theta$$

$$\int 64 \sin^3\theta\, d\theta$$

$u = \cos\theta$

$du = -\sin\theta\, d\theta$

$$-64\int (1-u^2)\, du$$

$$-64 \left[u - \frac{u^3}{3} \right]$$

$$-64\left(\cos\left(\sin^{-1}\left(\frac{x}{4}\right)\right) - \frac{\left(\cos\left(\sin^{-1}\left(\frac{x}{4}\right)\right)\right)^3}{3} \right)$$

$$-64\left[\left(\frac{\sqrt{16-x^2}}{4}\right) - \frac{\left(\frac{\sqrt{16-x^2}}{4}\right)^3}{3} \right]_0^{2\sqrt{3}}$$

$\sin(\theta) = x/4$