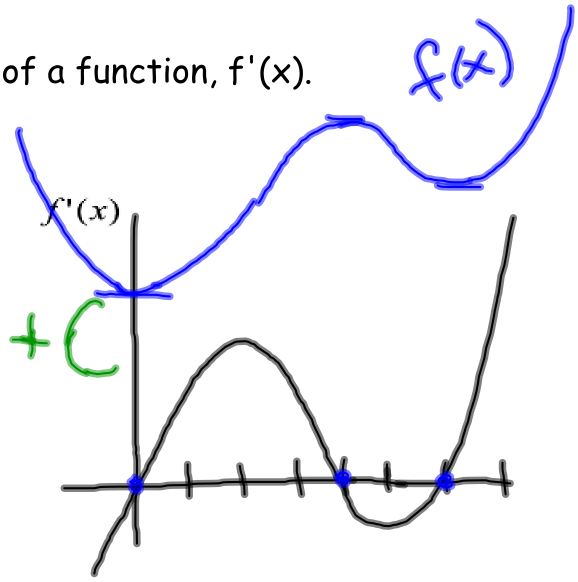
Math 152 Calculus and Analytic Geometry II

Sec 4.9 Antiderivatives

Suppose you are given the derivative of a function, f '(x).
Can you find the original function?

$$f'(x) = 4x^3 + 10x - 7$$

$$f(x) = x^4 + 5x^2 - 7x + C$$



Definition: A function F is called the antiderivative of f on an interval I if
$F'(x) = f(x)$ for all x in the interval I.

Much of our time in 152 will be spent learning how to find antiderivatives and what applications they have.

Can a function have more than one antiderivative?

Theorem: If $F(x)$ and $G(x)$ are antiderivatives of $f(x)$ then

$$F(x) = G(x) + C$$

for some C.

Theorem: If F is an antiderivative of f on an interval I, then all antiderivatives of f are of the form

$$F(x)+C$$

$f(x)=x^n$ $F(x)=\dfrac{x^{n+1}}{n+1}+C$

Examples of Common Antiderivatives:

$c\cdot f(x)$ $c\cdot F(x)+C$

$f(x)=\sqrt{x}=x^{1/2}$ $\frac{2}{3}\cdot x^{3/2}$

$f(x)=x$ $\frac{1}{2}x^2+C$

$f(x)=\frac{1}{x}$ x^{-1} $\cancel{\frac{x^0}{0}}$ $\rightarrow F(x)=\ln(x)+C$

$f(x)=x^2$ $\frac{1}{3}x^3+C$

$f(x)=e^x$ $F(x)=e^x+C$

$f(x)=x^3$ $\frac{x^4}{4}+C$

$f(x)=\frac{1}{1+x^2}$

$F(x)=\tan^{-1}(x)+C$

$f(x)$	$F(x)$
$\cos(x)$	$\sin(x)+C$
$\sin(x)$	$-\cos(x)+C$
$\sec^2(x)$	$\tan(x)+C$
$\sec(x)\tan(x)$	$\sec(x)+C$

Differential Equations

Find f(x) where $f'(x) = 3x^2 - 6x - 9$ $\quad f(1) = -5$

↑ use this to find C

Find the antiderivative that satisfies the condition.
Also graph the derivative, and use that to graph the function.

$$f(x) = x^3 - 3x^2 - 9x + C$$

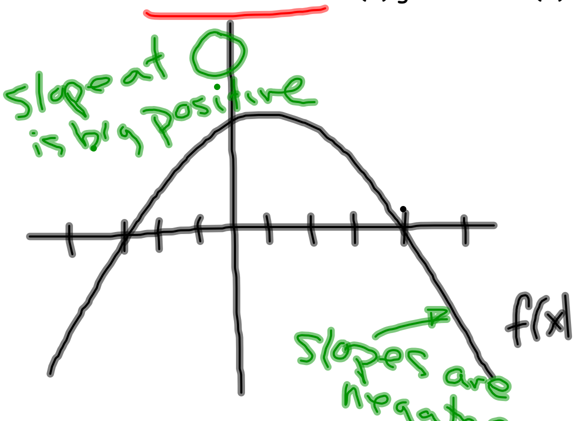
$$-5 = (1)^3 - 3(1)^2 - 9(1) + C$$

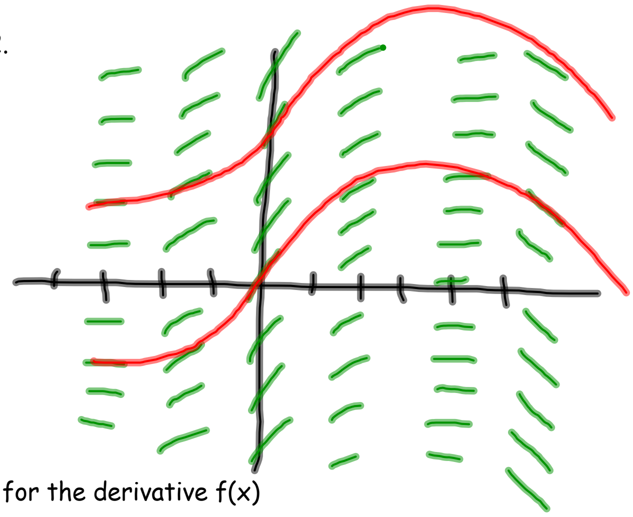
$$C = +6$$

$$\boxed{f(x) = x^3 - 3x^2 - 9x + 6}$$

Geometry of Antiderivatives.

Draw the antiderivative of f(x) given that F(0)=2.





Another way to do it if you are given an equation for the derivative f(x)
Use "Direction Fields"
Double click on f(x) on the left to change the derivative (slope) funciton.

http://www.math.ohio-state.edu/~maharry/GeoGebra/SlopeFieldGenerator02FromDaveMatthews.html

Rectilinear Motion: A ball is thrown upwards with a speed of 48 ft/sec from a cliff 400 feet high. Find its height after t seconds.

Start with a(t) = 32 (ft/sec)/sec.

$a(t) = -32$

$v(t) = -32t + C$ initial velocity

antiderivative of acceleration

$v(t) = -32t + 48$

$s(t) = -16t^2 + 48t + C$ initial height

position

$$\boxed{s(t) = -16t^2 + 48t + 400}$$

Practice exercises:

Find the most general antiderivative:

$f(x) = 1 - x^3 + 3x^5 - x^7$

$F(x) = x - \frac{x^4}{4} + \frac{3x^6}{6} - \frac{x^8}{8} + C$

$f(u) = \frac{u^3 - \sqrt{u}}{u^2}$

$f(u) = \frac{u^3}{u^2} - \frac{\sqrt{u}}{u^2} = u - u^{-3/2}$ $F(u) = \frac{u^2}{2} - \left(\frac{-2}{1}\right)u^{-1/2}$

$= \frac{u^2}{2} + 2u^{-1/2} + C$

$f(x) = 4(5x^2 - 3x + 2)^3(10x - 3)$

Must have been chain rule

$F(x) = (5x^2 - 3x + 2)^4 + C$

Problems to work on from Homework:

Find f.

$f'(x) = 1 - 6x \qquad f(0) = 8$

$$F(x) = x - \frac{6x^2}{2} + C$$

$$8 = 0 - \frac{6(0)^2}{2} + C$$

$$C = 8$$

$$\boxed{f(x) = x - 3x^2 + 8}$$