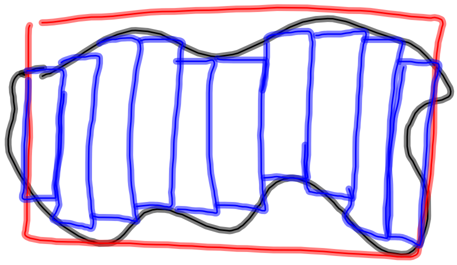
Math 152 Calculus and Analytic Geometry II

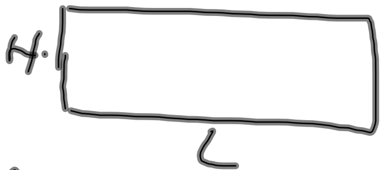
Sec 5.1 Areas and Distances

How can you find the area of a region with curvy sides?

How can you find the total distance travelled when the velocity is changing?



Approximate Area by lots of easy rectangles



Area = L · H

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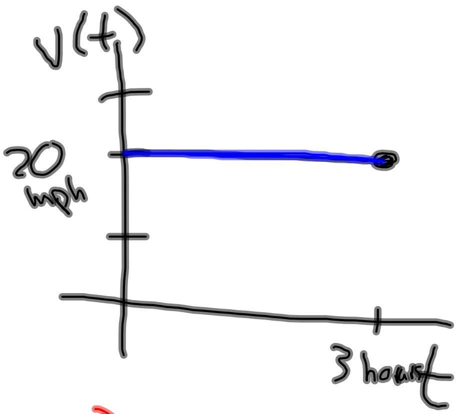
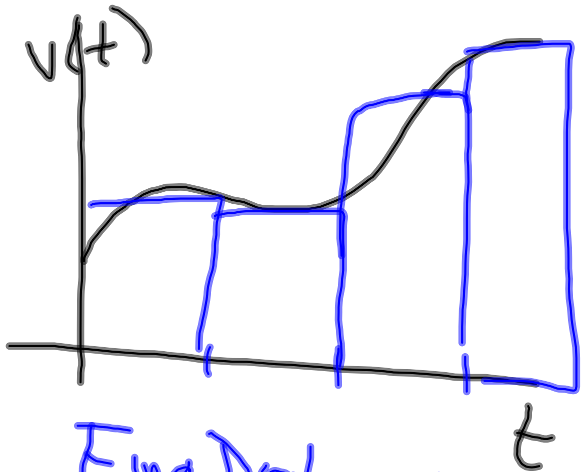




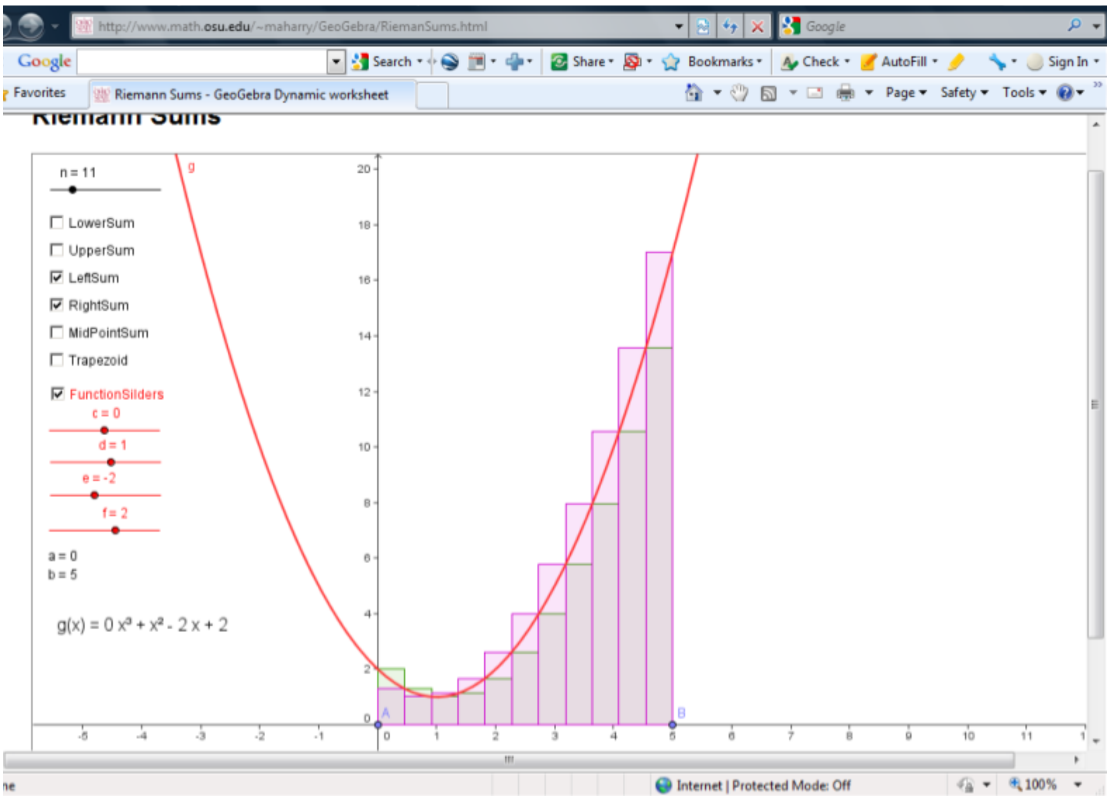
Distance = R · T
= 60 miles

Find Distance by approximating by rectangles

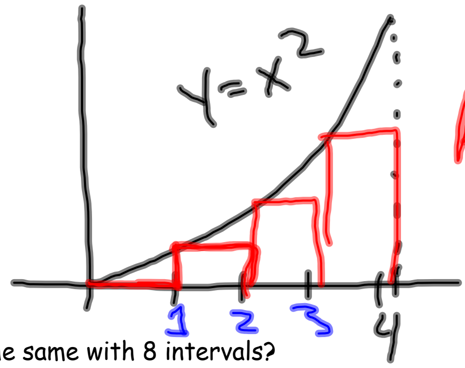
Start by plotting g(x)=x^2-2x+1 with n=1. Then change g(x) and change n, a and b.

http://www.math.ohio-state.edu/~maharry/GeoGebra/RiemanSums.html

http://mathworld.wolfram.com/RiemannSum.html



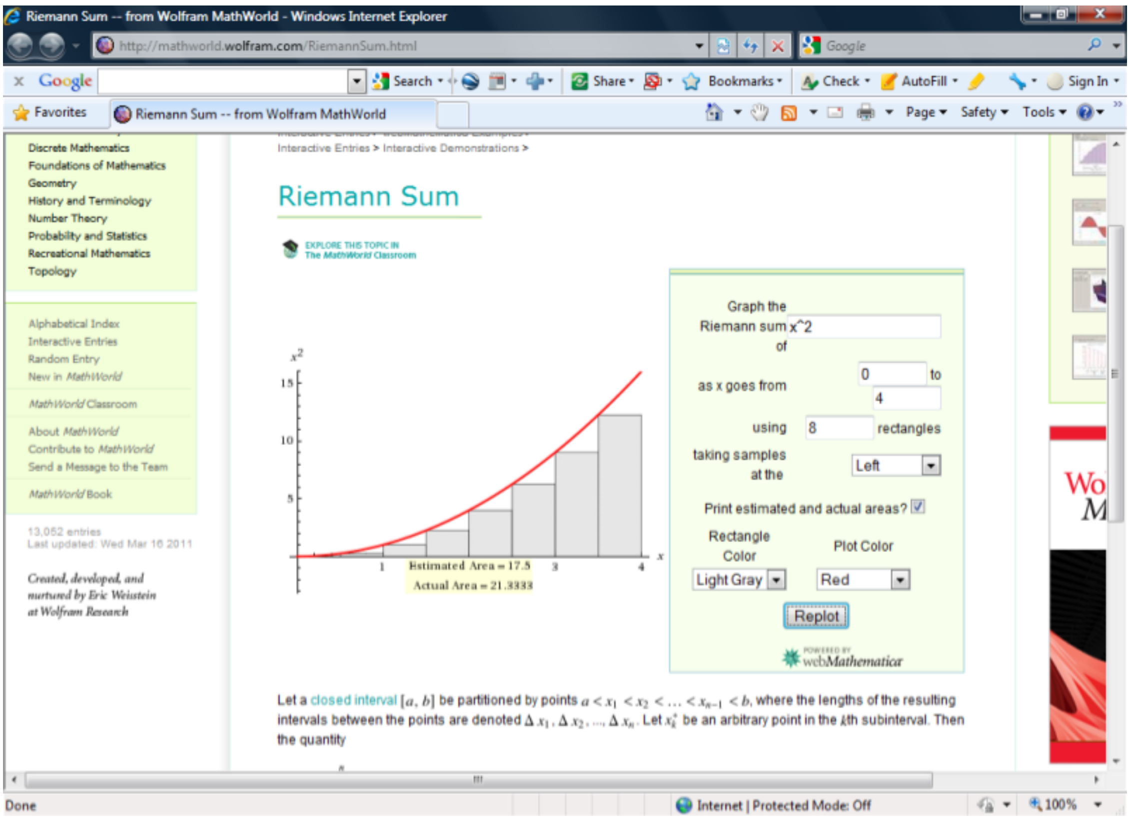
Try to do the same with y=x^2 over the interval from [0,4] with 4 intervals. Use the left endpoints.



$$\text{Area} = 1(0) + 1(1)^2 + 1(2)^2 + 1(3)^2 = 14$$

Try the same with 8 intervals?

Try it with the applet before to see what happens with many intervals...



Find the area under the curve y=x^2 on the interval [0,1] using n rectangles and the right endpoints.

n rectangles using "right endpoints"

Each rectangle we need
1) width
2) x-coordinate of right endpoint
3) height

guess little less than 1/2 maybe .4

Use this formula:

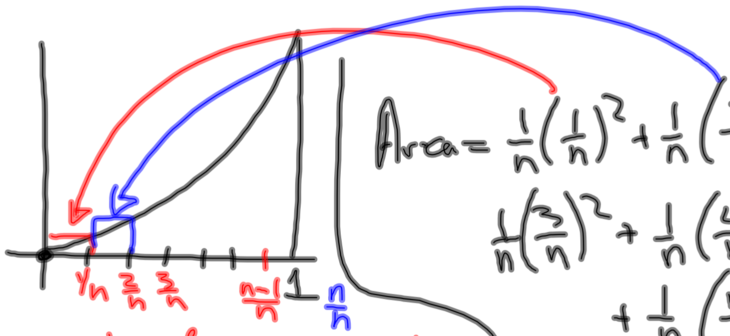
$$1^2+2^2+3^2+\cdots+n^2=\frac{n(n+1)(2n+1)}{6}$$

$$1^2+2^2+3^2=\frac{3(4)(7)}{6}$$

$$14=14$$

$$1^2+2^2+3^2+4^2=\frac{4(5)(9)}{6}$$

$$30 \qquad =30$$



$$\text{Area}=\frac{1}{n}\left(\frac{1}{n}\right)^2+\frac{1}{n}\left(\frac{2}{n}\right)^2+\frac{1}{n}\left(\frac{3}{n}\right)^2+\frac{1}{n}\left(\frac{4}{n}\right)^2+\cdots\cdots+\frac{1}{n}\left(\frac{n}{n}\right)^2$$

width of each rectangle $= 1/n$

x-coord of right endpoint =

heights are $\left(\frac{1}{n}\right)^2, \left(\frac{2}{n}\right)^2 \ldots$

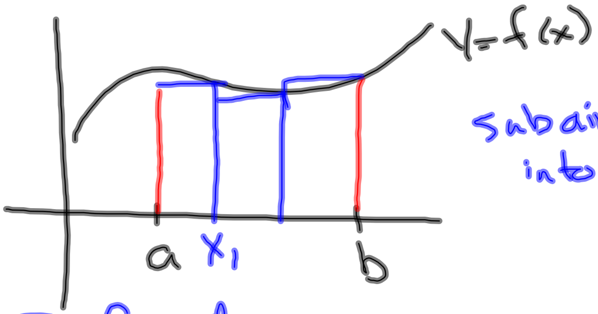
$$=\frac{1}{n^3}\left(1^2+2^2+3^2+\cdots+n^2\right)$$

$$=\frac{1}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right)$$

$$\text{Exact Area}=\lim_{n\to\infty}\frac{n(n+1)(2n+1)}{6n^3}$$

$$=\lim_{n\to\infty}\frac{2n^3+\cdots\cdots}{6n^3}=1/3.$$

Now we will try the same idea with a general function y=f(x) on the interval [a,b] using n equal intervals.



subdivide interval into n rectangles

To find Area we need

width of rectangles $= \frac{b-a}{n}$

right endpoint $= a + \left(\frac{b-a}{n}\right)$

$a + 2\left(\frac{b-a}{n}\right)$

$a + 3\left(\frac{b-a}{n}\right)$

$\vdots$

i^{th} rectangle $a + i\left(\frac{b-a}{n}\right)$

n^{th} rectangle $a + n\left(\frac{b-a}{n}\right)$

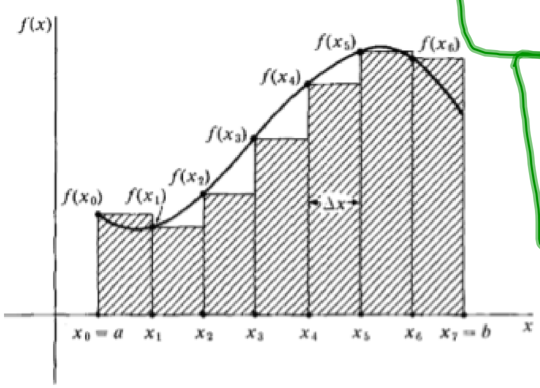
Definition: The Area A if the region S that lies under the graph of the continuous function f(x) is the limit of the sum of the areas of the approximating rectangles.

$$A = \lim_{n \to \infty}\left[f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_n)\Delta x\right]$$

$$\boxed{\Delta x = \frac{b-a}{n}}$$

$$\boxed{x_i = a + i\left(\frac{b-a}{n}\right)}$$

using right endpoints



Another Riemann Sum Plotter
http://mathworld.wolfram.com/RiemannSum.html

Sigma Notation using Right endpoints of equal intervals (Technically, the intervals don't have to be equal, but they do have to all get shorter and shorter with width going to zero)

$$A = \lim_{n \to \infty} \left[f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + \cdots + f(x_n)\Delta x \right]$$

$$A = \lim_{n \to \infty} \left[\sum_{i=1}^{n} f(x_i)\Delta x \right]$$

$\sum_{i=1}^{n}$ means summation of terms from 1 to n.

Also, the same can be done with any 'sample point' from each interval. The limit will be the same since f(x) is continuous.

$$A = \lim_{n \to \infty} \left[\sum_{i=1}^{n} \underbrace{f(x_i^*)\Delta x}_{\text{Area of rectangle}} \right]$$

take more and more rectangles

add them all up

Distance problem.

Suppose you record the speed (in miles per minute) you are travelling every 10 minutes while you fly your jet for an hour.

Time	0	10	20	30	40	50	60
Velocity	5	8	11	14	17	20	23

How far did you travel? $\Delta x = 10$ minutes

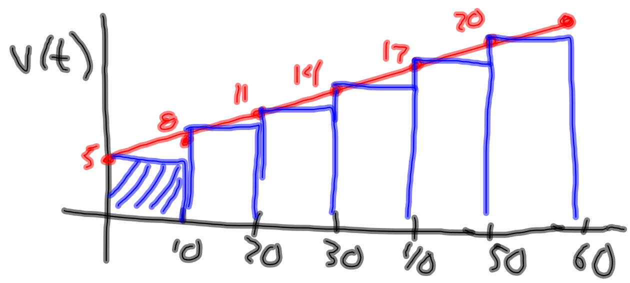
$D = 10(5) + 10(8) + 10(11) + 10(14) + 10(17) + 10(20)$

$= \cancel{550}$ 750 miles

How could you be more accurate?

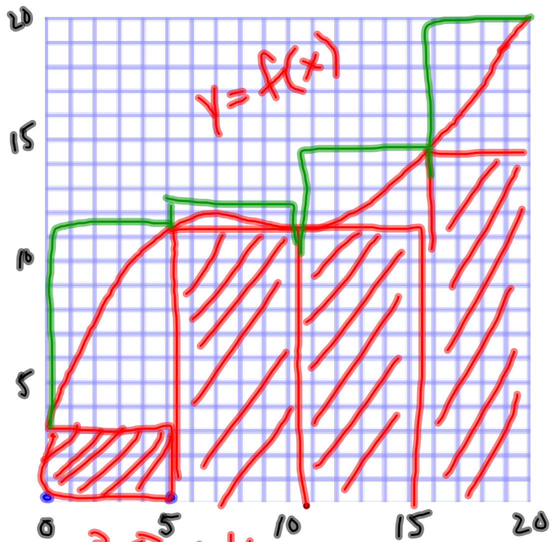
What might be a function v(t) for your velocity?

http://mathworld.wolfram.com/RiemannSum.html



Examples from the Homework

Using the graph as a function f(x), use ~~5~~ 4 rectangles to find a lower red and upper estimate for the area under the curve on the interval [0,20] Draw the rectangles you use. green



Area =

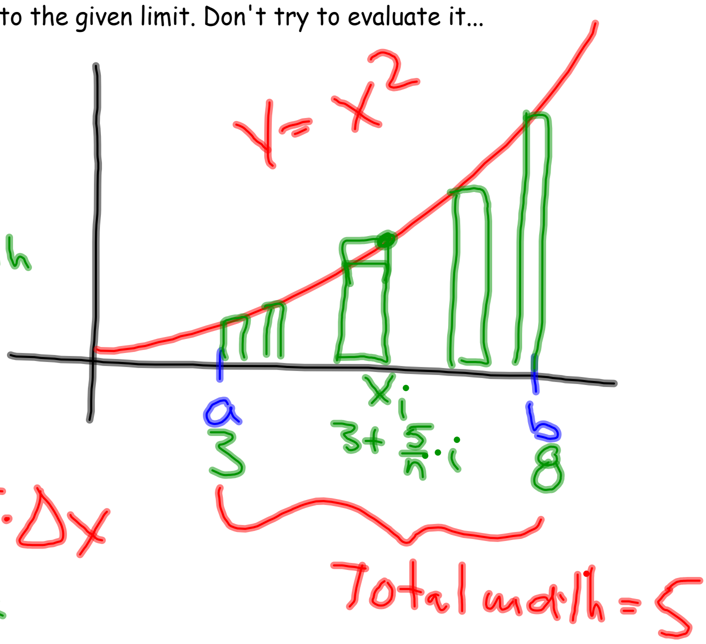
$3.5 + 11.5 + 11.5 + 14.5 =$

#13 Oil leaked from a tank at a rate of r(t) liters per hour. The values are given in the table in 2-hour intervals. Find the upper and lower estimates for the amount of oil that leaked.

Time (hours)	0	2	4	6	8	10
r(t) (liters/hour)	8.7	7.6	6.8	6.2	5.7	5.3

Determine a region whose area is equal to the given limit. Don't try to evaluate it...

$$A = \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\underbrace{\left(3+\frac{5i}{n}\right)^2}_{\text{height of each rectangle}}\ \underbrace{\frac{5}{n}}_{\text{width}}\right)\right]$$



$$\lim_{n\to\infty}\sum_{i=1}^{n}\underbrace{(\qquad)^2}_{\text{height}}\cdot\Delta x$$