Math 152 Calculus and Analytic Geometry II

Sec 5.2 The Definite Integral

Definition of a Definite Integral:
If f(x) is a continuous function defined on [a,b], we divide the interval into n subintervals of equal width $\Delta x = \frac{b-a}{n}$

We let $x_0(=a), x_1, x_2, \cdots, x_{n-1}, x_n(=b)$ be the endpoints of the subintervals and we let $x^*_1, x^*_2, \cdots, x^*_{n-1}, x^*_n$
be any sample points in each of the subintervals,
(whew... take a breath...)

Then the <u>definite integral of f from a to b</u> is

$$\int_a^b f(x)dx = \lim_{n\to\infty}\sum_{i=1}^{n} f(x_i^*)\Delta x$$

Notes:

- Since f(x) is continuous, the limit always exists.
- The limit does not depend on the sample points chosen (Since f(x) is continuous, any sample point in a small interval must have almost the same y-value)
- The limit also exists even if the function f(x) has a finite number of jumps, holes or breaks.
- But it <u>does not always exist</u> if there is a <u>vertical asymptote in the interval</u> !!!!
- The definite integral $\int_a^b f(x)dx$ is a number. It does not depend on 'x'.

- The sum is called a "Reimann sum" after Bernhard Riemann (1822-1866)
- If the function is always positive, then the sum can be thought of as the area under the curve.
- If the function is sometimes positive and sometimes negative, we'll have to rethink that...

http://en.wikipedia.org/wiki/Bernhard_Riemann

Express the following Limit as a definite integral on the interval $[0, \pi]$

$$\lim_{n \to \infty} \sum_{i=1}^{n} \left(1 + 3x_i^{\,2}\right) Sin(x_i) \Delta x$$

How can we evaluate these limits?

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}$$

$$\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{i=1}^{n} i^3 = \left(\frac{n(n+1)}{2}\right)^2$$

http://www.math.utah.edu/~cherk/ccli/bob/sums.html

Evaluate the following using rules for summations. Use the interval [2,8]

$$f(x) = x^2 - 5x + 2$$

$$\int_2^8 x^2 - 5x + 2dx = \lim_{n \to \infty} \sum_{i=1}^{n} \left(x_i^2 - 5x_i + 2\right) \frac{b-a}{n}$$

Evaluate the following definite integral. (Think before you try some ugly limits...)

$$\int_{-4}^{4}\left(2+\sqrt{16-x^2}\right)dx = \underset{n\to\infty}{Lim}\sum_{i=1}^{n}\left(2+\sqrt{16-x_i^{\,2}}\right)\frac{b-a}{n}$$

Another Riemann Sum Plotter
http://mathworld.wolfram.com/RiemannSum.html

Properties of a Definite Integral

$$\int_a^b c\,dx = c(b-a)$$

$$\int_a^b \left(f(x)+g(x)\right)dx = \int_a^b f(x)dx + \int_a^b g(x)dx$$

$$\int_a^b c\left(f(x)\right)dx = c\int_a^b f(x)dx$$

$$\int_a^b \left(f(x)\right)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$$

Notice that these properties are true if a<b.

But also if a=b or if a<b

What does that mean?

Comparison Theorems for Definite Integrals

If $m \leq f(x) \leq M$ for all $a \leq x \leq b$ then,

$$m(b-a) \leq \int_a^b (f(x))dx \leq M(b-a)$$

Evaluate the following using rules for summations. Use the interval [0,4]

$$\int_0^4 x^3 - 6x^2 dx$$

Evaluate the following using rules for summations. Use the interval [-3,5]

$$\int_{-3}^{5} 3x^2 - 4x dx$$