

Math 152 Calculus and Analytic Geometry II

Sec 5.3 The Fundamental Theorem of Calculus

A couple examples before we start:

Use the Limit Definition to find: $\int_a^b x dx =$

.

A couple examples before we start:

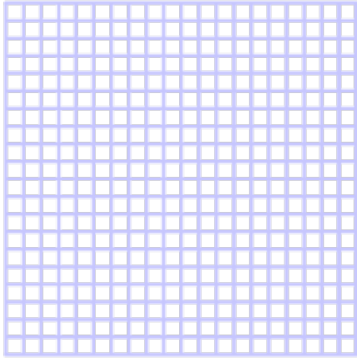
Use the Limit Definition to find: $\int_a^b x^2 dx =$

We define a new function of x with the variable in the upper limit of a definite integral.

$$g(x) = \int_0^x f(t) dt$$

Consider $f(t)$ to be velocity and $g(x)$ to be distance traveled after x seconds.

The book calls it the "area so far".
It is also called an "accumulation function".



What can we say about the derivative of $g(x)$?

Consider the difference between $g(x+h)$ and $g(x)$

What is the limit definition of the derivative of $g(x)$?

The Fundamental Theorem of Calculus Part I

$$g(x) = \int_0^x f(t) dt$$

If f is continuous on $[a,b]$ then $g(x)$ is continuous on $[a,b]$ and differentiable on (a,b) and

$$g'(x) = f(x)$$

$$\frac{d}{dx} \int_a^x f(t) dt = f(x)$$

Examples:

$$g(x) = \int_3^x \sqrt{t} dt$$

$$h(x) = \int_{-\pi}^x \cos(t) dt$$

$$h(x) = \int_{-1}^x u^3 \ln(u+5) du$$



$$g(x) = \int_x^3 \sqrt{t} dt$$

$$g(x) = \int_3^{2x} \sqrt{t} dt$$

$$g(x) = \int_0^{x^2} \sqrt{t} dt$$

The Fundamental Theorem of Calculus Part II

If f is continuous on $[a,b]$ then

$$\int_a^b f(t) dt = F(b) - F(a)$$

where F is any antiderivative of f , that is $F'(x) = f(x)$.

Proof: We know one antiderivative $g(x) = \int_a^x f(t) dt$

Any other antiderivative must be $F(x) = g(x) + C$

Plug in $x=a$

Calculate $F(b) - F(a)$

One example:

If $v(t)$ is velocity of an object, $v(t) = s'(t)$, where $s(t)$ is the position function.

From examples we have done, the area under the curve of $v(t)$ is equal to the distance travelled.

FTC Part I: Take a function, integrate it and then take the derivative.

FTC Part II: Take a function, find its derivative and then integrate.

Evaluate the following using rules for summations. Use the interval [2,8]

$$f(x) = x^2 - 5x + 2$$

$$\int_2^8 x^2 - 5x + 2 dx = \lim_{n \rightarrow \infty} \sum_{i=1}^n (x_i^2 - 5x_i + 2) \frac{b-a}{n}$$

Use FTC Part II to evaluate the integrals

$$\int_2^5 (4x + 3) dx$$

$$\int_{-\pi}^{\pi} (\sin(x) - 5x) dx$$

$$\int_{-2}^4 \frac{3}{x^2} dx$$

$$\int_2^4 \frac{3}{x^2} dx$$

A manufacturing company owns a major piece of equipment that depreciates at the continuous rate $f=f(t)$, where t is the time in months since the last overhaul. Because a fixed cost is incurred each time the machine is overhauled, the company wants to determine the optimal time T (in months) between overhauls.

a) What does $\int_0^t f(s) ds$ represent?

b) Let $C(t) = \frac{1}{t} \left[A + \int_0^t f(s) ds \right]$ What does $C(t)$ mean and why do you want to minimize it?

c) Show that $C(t)$ has a minimum value at the numbers $t=T$ where $C(T)=f(T)$.

Attachments



FundamentalTheoremCalculus.ggb