Math 152 Calculus and Analytic Geometry II

Sec 5.3 The Fundamental Theorem of Calculus

A couple examples before we start:

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We define a new function of x with the variable in the upper limit of a definite integral.

$$g(x) = \int_0^x f(t)dt$$

Consider f(t) to be velocity and g(x) to be distance traveled after x seconds.

The book calls it the "area so far".
It is also called an "accumulation function".

What can we say about the derivative of g(x)?

Consider the difference between g(x+h) and g(x)

What is the limit definition of the derivative of g(x)?

The Fundamental Theorem of Calculus Part I

$$g(x) = \int_0^x f(t)dt$$

If f is continuous on [a,b] then g(x) is continuous on [a,b] and differentiable on (a,b) and $g'(x) = f(x)$

$$\frac{d}{dx}\int_a^x f(t)dt = f(x)$$

Examples:

$$g(x) = \int_3^x \sqrt{t}\,dt$$

$$h(x) = \int_{-\pi}^x Cos(t)dt$$

$$h(x) = \int_{-1}^x u^3 \ln(u+5)du$$

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$$g(x)=\int_x^3 \sqrt{t}\,dt$$

$$g(x)=\int_3^{2x} \sqrt{t}\,dt$$

$$g(x)=\int_0^{x^2} \sqrt{t}\,dt$$

The Fundamental Theorem of Calculus Part II

If f is continuous on [a,b] then

$$\int_a^b f(t)dt = F(b)-F(a)$$

where F is any antiderivative of f , that is F'(x) = f(x).

Proof: We know one antiderivative $g(x)=\int_a^x f(t)dt$

Any other antiderivative must be $F(x)=g(x)+C$

Plug in x=a

Calculute F(b) - F(a)

One example:

If $v(t)$ is velocity of an object, $v(t) = s'(t)$, where $s(t)$ is the position function.

From examples we have done, the area under the curve of $v(t)$ is equal to the distance travelled.

FTC Part I: Take a function, integrate it and then take the derivative.

FTC Part II: Take a function, find its derivative and then integrate.

Evaluate the following using rules for summations. Use the interval [2,8]

$$f(x) = x^2 - 5x + 2$$

$$\int_2^8 x^2 - 5x + 2dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(x_i^{\ 2} - 5x_i + 2\right)\frac{b-a}{n}$$

Use FTC Part II to evaluate the integrals

$$\int_2^5 (4x+3)dx$$

$$\int_{-\pi}^{\pi} (Sin(x) - 5x)dx$$

$$\int_{-2}^{4} \frac{3}{x^2}\, dx$$

$$\int_{2}^{4} \frac{3}{x^2}\, dx$$

A manufacturing company owns a major piece of equipment that depreciates at the continuous rate f=f(t), where t is the time in months since the last overhaul. Because a fixed cost is incurred each time the machine is overhauled, the company wants to determine the optimal time T (in months) between overhauls.

a) What does $\int_0^t f(s)ds$ represent?

b) Let $C(t) = \frac{1}{t}\left[A + \int_0^t f(s)ds \right]$ What does C(t) mean and why do you want to minimize it?

c) Show that C(t) has a minimum value at the numbers t=T where C(T)=f(T).

Attachments

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