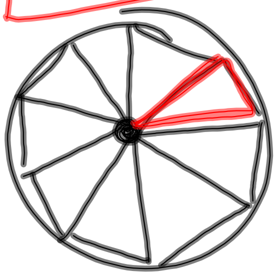
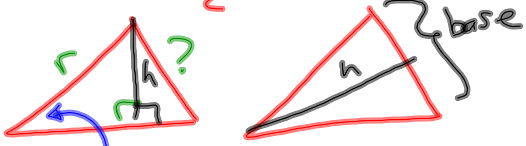
HW 4.9 + 5.1

$\boxed{5.1\ \#7}$

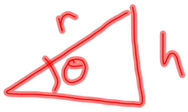


n triangles

Area $= \frac{1}{2} b \cdot h$



$\theta = \frac{2\pi}{n}$



$\sin(\theta) = \frac{h}{r}$

$$A_n = \frac{1}{2} n r^2 \sin\left(\frac{2\pi}{n}\right)$$

Part 2

use $A_n = \frac{1}{2} n r^2 \sin\left(\frac{2\pi}{n}\right)$

to Find

$$\lim_{n\to\infty} \frac{1}{2} n r^2 \sin\left(\frac{2\pi}{n}\right)$$

Know

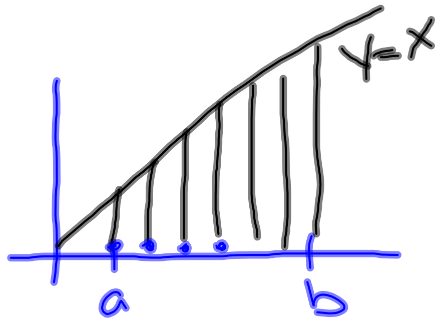
$$\lim_{x\to 0} \frac{\sin(x)}{x} = 1$$

$= \pi r^2$

$$\boxed{\lim_{\frac{2\pi}{n}\to 0} \frac{\sin\left(\frac{2\pi}{n}\right)}{\left(\frac{2\pi}{n}\right)} = 1}$$

Math 152 Calculus and Analytic Geometry II

Sec 5.3 The Fundamental Theorem of Calculus



A couple examples before we start:

Use the Limit Definition to find: $\int_a^b x\,dx =$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\left(a+\left(\frac{b-a}{n}\right)i\right)}_{\text{height}} \underbrace{\left(\frac{b-a}{n}\right)}_{\text{width}}$$

$\Delta x = \frac{b-a}{n}$

$x_i = a + \left(\frac{b-a}{n}\right)i$

$$= \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \sum_{i=1}^{n} \left(a+\left(\frac{b-a}{n}\right)i\right)$$

$$= \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \left[\sum_{i=1}^{n} \left(a+\left(\frac{b-a}{n}\right)i\right)\right]$$

$$= \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \left[\sum_{i=1}^{n} a + \sum_{i=1}^{n} \left(\frac{b-a}{n}\right) i\right]$$

$$= \lim_{n\to\infty} \left(\frac{b-a}{n}\right) \left[n\cdot a + \left(\frac{b-a}{n}\right)\left(\frac{n(n+1)}{2}\right)\right]$$

$$= \lim_{n\to\infty} \frac{(b-a)(a)n}{n} + \frac{(b-a)^2}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= (b-a)(a) + \frac{(b-a)^2}{2}$$

$(b-a)(a) + \frac{(b-a)^2}{2}$

$\frac{2(ab-a^2)}{2} + \frac{b^2-2ab+a^2}{2}$

$= \frac{b^2-a^2}{2} = \boxed{\frac{b^2}{2} - \frac{a^2}{2}}$

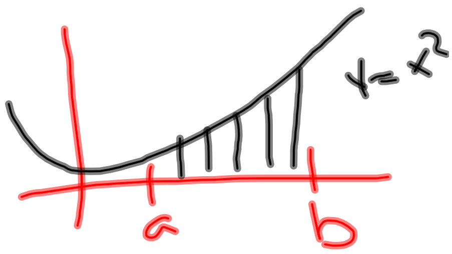
$\int_a^b x\,dx$ $\quad \frac{x^2}{2}$ plug in endpoints

$F(b)-F(a)$

$\frac{b^2}{2} - \frac{a^2}{2}$

A couple examples before we start:

Use the Limit Definition to find: $\int_a^b x^2 dx =$



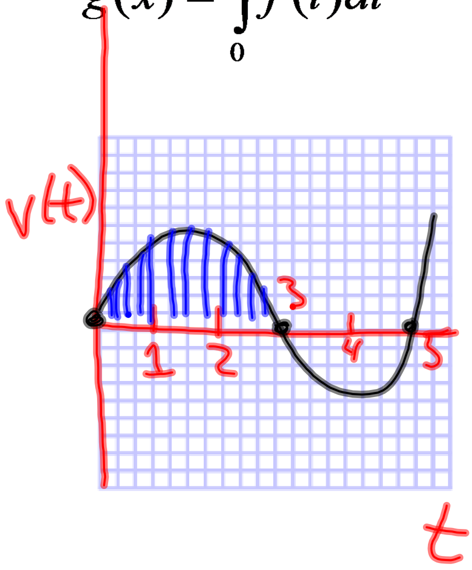
$$\lim_{n\to\infty} \sum_{i=1}^{n} \underbrace{\left(a+\left(\frac{b-a}{n}\right)i\right)^2}_{(x_i)^2 = \text{height}} \underbrace{\left(\frac{b-a}{n}\right)}_{\Delta x = \text{math.}}$$

algebra

$\frac{b^3}{3} - \frac{a^3}{3}$

We define a new function of x with the variable in the upper limit of a definite integral.

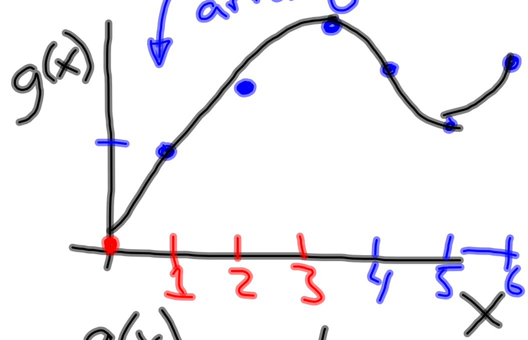
$$g(x) = \int_0^x f(t)\,dt$$



Consider f(t) to be velocity and g(x) to be distance traveled after x seconds.

The book calls it the "area so far". It is also called an "accumulation function".

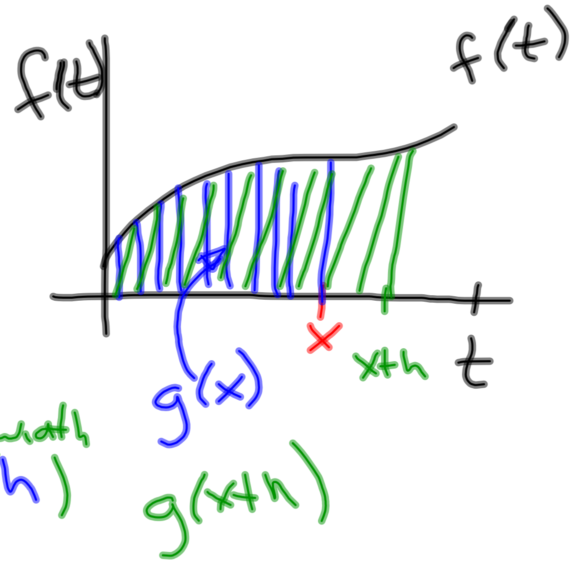
height = area of



g(x) = distance travelled fm t=0 to t=x

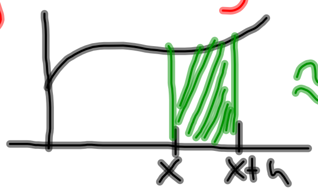
What can we say about the derivative of g(x)?

$$g(x) = \int_0^x f(t)\,dt$$



Consider the difference between g(x+h) and g(x)

$g(x+h) - g(x)$



height · width

$\approx (f(x)) \cdot (h)$

What is the limit definition of the derivative of g(x)?

$$g'(x) = \lim_{h \to 0} \frac{g(x+h) - g(x)}{h} = \frac{f(x) \cdot h}{h} = f(x)$$

The Fundamental Theorem of Calculus Part I

$$g(x) = \int_{\cancel{0}\; a}^{x} f(t)\,dt$$

If f is continuous on [a,b] then g(x) is continuous on [a,b] and differentiable on (a,b) and $g'(x) = f(x)$

$$\frac{d}{dx}\int_{a}^{x} f(t)\,dt = f(x)$$

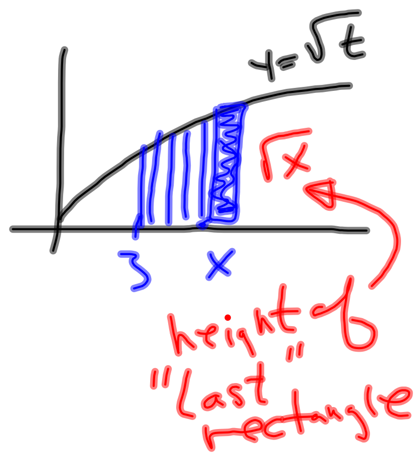
$$g'(x) = \frac{d}{dx}\,g(x) = \underbrace{\frac{d}{dx}\left(\int_{a}^{x} f(t)\,dt\right)}_{\text{rate of change of accumulation function}} = f(x)$$

= height of function at x.

Examples:

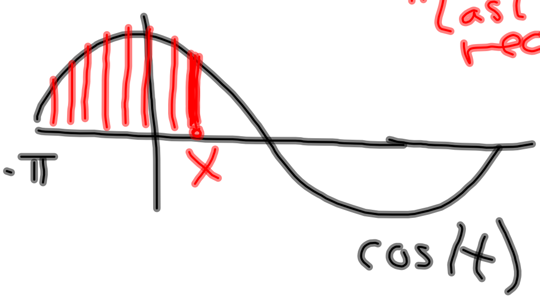
$$g(x) = \int_{3}^{x} \sqrt{t}\,dt \qquad g'(x) = \sqrt{x}$$

$$\int_{0}^{x} \sqrt{t}\,dt = \sqrt{x}$$



$$h(x) = \int_{-\pi}^{x} Cos(t)\,dt$$

$$h'(x) = Cos(x)$$



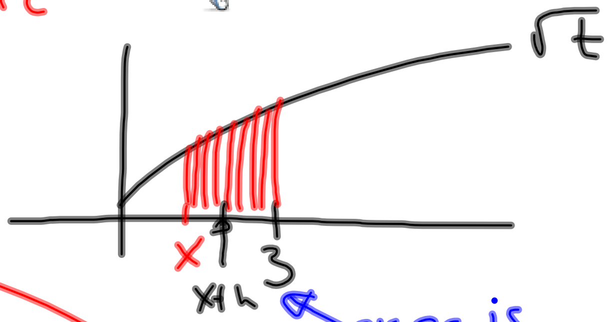
$$h(x) = \int_{-1}^{x} u^{3}\ln(u+5)\,du$$

$$h'(x) = x^{3}\cdot \ln(x+5)$$

FundamentalTheoremCalculus.ggb

$g(x)=\int_x^3 \sqrt{t}\,dt = \int_3^x \sqrt{t}\cdot dt$

$g'(x) = -\sqrt{x}$



← area is decreasing so rate is negative

$g(x)=\int_3^{2x}\sqrt{t}\,dt$

$g(u)=\int_3^u \sqrt{t}\,dt$

$g(x)=\int_3^{(2x)}\sqrt{t}\,dt$

$\boxed{g'(x)=\sqrt{2x}\cdot 2}$

chain rule

$g(x)=\int_0^{x^2}\sqrt{t}\,dt$

$g'(x)=\sqrt{x^2}\cdot 2x$

The Fundamental Theorem of Calculus Part II

If f is continuous on [a,b] then

$$\int_a^b f(t)dt = F(b)-F(a)$$

where F is any antiderivative of f , that is F'(x) = f(x).

Earlier.

$\int_a^b x\,dx$

$f(x)=x$

$F(x)=\frac{x^2}{2}$

$= \frac{b^2}{2}-\frac{a^2}{2}$

Proof: We know one antiderivative $g(x)=\int_a^x f(t)dt$

Any other antiderivative must be $F(x)=g(x)+C$

g(x) is an antiderivative since $g'(x)=f(x)$

Plug in x=a

$F(a)=\int_a^a f(t)dt + C$

Calculute F(b) - F(a)

$F(a)=0+C$ i.e. $C=F(a)$

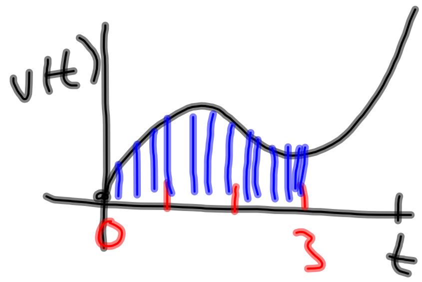
$F(b)-F(a)=\left(\int_a^b f(t)dt+F(a)\right)-F(a)$

$\boxed{F(b)-F(a)=\int_a^b f(t)dt}$

One example:

If v(t) is velocity of an object, v(t) = s'(t), where s(t) is the position function.

From examples we have done, the area under the curve of v(t) is equal to the distance travelled.



$$\int_a^b f(x)\,dx = F(b)-F(a)$$

$$\underbrace{\int_0^3 v(t)\,dt}_{\text{Add up all of short distances}} = s(3)-s(0)$$

position at end minus position at start = distance

FTC Part I: Take a function, integrate it and then take the derivative.

$$\frac{d}{dx}\left[\underbrace{\int_a^x f(t)\,dt}_{\text{accumulation function}}\right] = f(x)$$

FTC Part II: Take a function, find its derivative and then integrate.

$$F(x)$$

$$\int_a^b f(x)\,dx = F(b)-F(a)$$

Evaluate the following using rules for summations. Use the interval [2,8]

$$f(x) = x^2 - 5x + 2$$

$$\int_2^8 x^2 - 5x + 2dx = \lim_{n\to\infty} \sum_{i=1}^{n} \left(x_i^{\,2} - 5x_i + 2\right)\frac{b-a}{n}$$

$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(2+\frac{6}{n}i\right)^2 - 5\left(2+\frac{6}{n}i\right)+2\right]\left(\frac{6}{n}\right)$$

lots of algebra

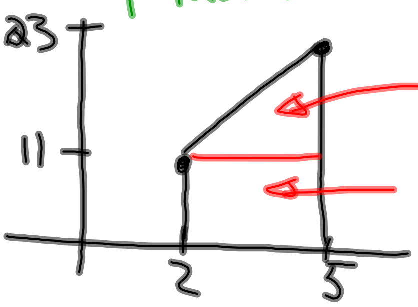
$$\int_2^8 x^2 - 5x + 2dx = F(8) - F(2)$$

$$F(x) = \frac{x^3}{3} - \frac{5x^2}{2} + 2x$$

Use FTC Part II to evaluate the integrals

$$\int_2^5 (4x+3)dx = 2x^2 + 3x\Big]_2^5 = \left(2(5)^2 + 3(5)\right) - \left[2(2)^2 + 3(2)\right] = 65 - 14 = \boxed{51}$$

Find antiderivative



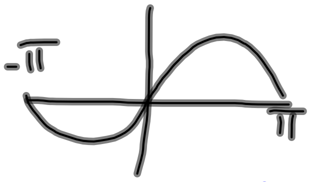
$\frac{1}{2}(3)(12)$

$11 \cdot 3$ = 51

$$\int_{-\pi}^{\pi}(Sin(x)-5x)dx = -Cos(x)-\frac{5x^2}{2}\Big]_{-\pi}^{\pi}$$

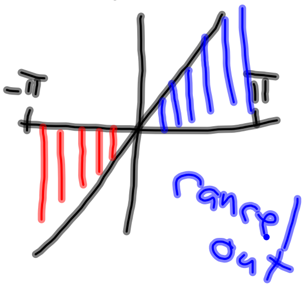
$$\int_{-\pi}^{\pi} Sin(x)\cancel{dx} - \int_{-\pi}^{\pi} 5x\,dx$$

plug in endpoints



$$=\left[-\cos(\pi)-\frac{5\pi^2}{2}\right]-\left[-\cos(-\pi)-\frac{5(-\pi)^2}{2}\right]$$

$$=0$$



$$\int_{-2}^{4}\frac{3}{x^2}dx = -\frac{3}{x}\Big]_{-2}^{4} = \left(\frac{-3}{4}\right)-\left(\frac{-3}{-2}\right)$$

$$f(x)=\frac{3}{x^2}=3x^{-2}$$

$$F(x)=-3x^{-1}$$

$$=-\frac{3}{4}-\frac{3}{2}=\frac{-9}{4}$$

Function must be continuous on [a,b]

Area should be positive not $-\frac{9}{4}$

$$\int_{2}^{4}\frac{3}{x^2}dx = \frac{-3}{4}+\frac{3}{2}$$

$$=3/4$$ ok

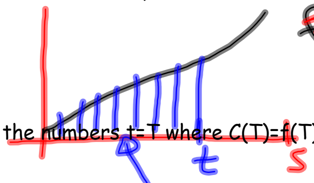
A

A manufacturing company owns a major piece of equipment that depreciates at the continuous rate f=f(t), where t is the time in months since the last overhaul. Because a fixed cost is incurred each time the machine is overhauled, the company wants to determine the optimal time T (in months) between overhauls.

~~f(s)~~ f(s)

\$/day in value "lost"

a) What does $\int_0^t f(s)ds$ represent?



c) Show that C(t) has a minimum value at the numbers t=T where C(T)=f(T).

A = cost to overhaul machine

Area = $\int_0^t f(s)ds$ = Total Depreciation.

b) Let $C(t)=\frac{1}{t}\left[A+\int_0^t f(s)ds\right]$

What does C(t) mean and why do you want to minimize it?

Total Cost of repairing then running for t days

$\boxed{C(t) = \text{avg cost per day}}$

c) Show that C(t) has a minimum value at the numbers t=T where C(T)=f(T).

Attachments

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