Math 152 Calculus and Analytic Geometry II

Sec 5.4 Indefinite Integrals and the Net Change Theorem

We need a notation for antiderivatives now that we have the Fundamental Theorem Part I and II

The notation $\int f(x)dx$ is used for an antiderivative of f(x) and is called an <u>indefinite integral</u>

Notes:
The indefinite integral represents a family of functions (one for each +C)

The definite integral (with endpoints [a,b]) is a <u>number</u> (sum of rectangles)

$$\int x^2 dx = \qquad\qquad \int_1^4 x^2 dx =$$

We adopt the convention that when a formula for a general indefinite integral is given, it is only valid on an interval.

For instance, we will write $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$

even though the general antiderivative is $F(x) = \begin{cases} -\frac{1}{x} + C_1 & if \quad x < 0 \\ -\frac{1}{x} + C_2 & if \quad x \geq 0 \end{cases}$

Try Some examples:

$$\int 4x - 5x^3 - 2\sqrt{x}dx$$

$$\int\left(\frac{4x - x^3}{x^2}\right)dx$$

$$\int(\sec\theta\tan\theta - \cos\theta)d\theta$$

Evaluate the following definite integrals using Part II of FTC.

$$\int_1^5 7 + 3t - t^2 dt =$$

$$\int_1^4 \frac{7x - \sqrt{x}}{x^2}dx =$$

The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x)dx = F(b) - F(a)$$

If Q(t) is the quantity of items stored in a warehouse, Q'(t) is the rate of change of the quantity at time t.
Find the total net change of quantity over the period from t=4 to t=8.

Suppose we know that $Q'(t) = 15 - 3t$ in units per day. Find the net change in stored items over the 4 day-period.

http://www.calculusapplets.com/accumulation.html

Suppose we know that $Q'(t) = 15 - 3t$ in units per day.

Estimate the net change in stored items over the 4-day period from t=4 to t=8

The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x)dx = F(b) - F(a)$$

If P(t) is the population of a certain city, P'(t) is the rate of change of the population per year at time t. Find the total net change of quantity over the period from t=5 to t=10.

Suppose we know that $P'(t) = 500\sqrt{t}$ in people per year. Find the net population change over the 5 year period.

The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x)dx = F(b) - F(a)$$

A particle moves along a straight path so that the velocity at time t is given by

$v(t) = 5 + 4t - t^2$ measured in feet per second.

Draw the velocity graph. Find the displacement of the particle from t=0 to t=7.

A particle moves along a straight path so that the velocity at time t is given by

$v(t) = 5 + 4t - t^2$ measured in feet per second.

Find the total distance traveled by the particle.

http://www.calculusapplets.com/eqofmotion.html