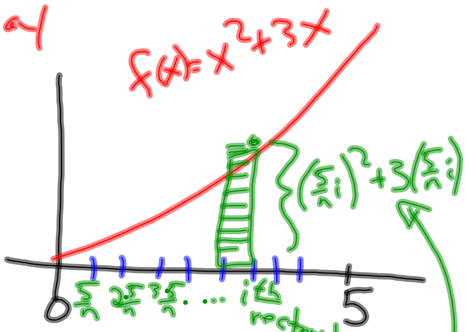
Quiz from Thursday

$$\int_0^5 x^2+3x\,dx$$



$$= \lim_{n\to\infty} \sum_{i=1}^{n} \left[\left(\frac{5}{n}i\right)^2 + 3\left(\frac{5}{n}i\right)\right]\cdot\left(\frac{5}{n}\right)$$

width $= \frac{5}{n}$

$x_i = \frac{5}{n}\cdot i$

right endpoint of ith interval $= \frac{5}{n}i$

height of ith rectangle

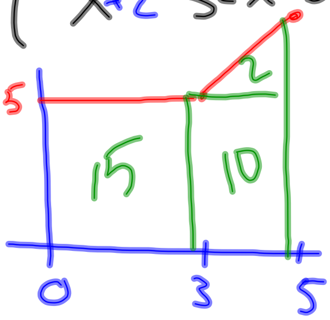
$$= \lim_{n\to\infty} \left(\frac{5}{n}\right)^3 \sum_{i=1}^{n} i^2 + 3\left(\frac{5}{n}\right)^2 \sum_{i=1}^{n} i$$

$$= \lim_{n\to\infty} \frac{5^3}{n^3}\left(\frac{n(n+1)(2n+1)}{6}\right) + \frac{3\cdot 5^2}{n^2}\left(\frac{n(n+1)}{2}\right)$$

$$= \boxed{\frac{5^3\cdot 2}{6} + \frac{3\cdot 5^2}{2}}$$

$$\int_0^5 g(x)\,dx \qquad g(x) = \begin{cases} 5 & 0<x<3 \\ x+2 & 3<x<5 \end{cases}$$

$$\int_0^3 g(x)\,dx + \int_3^5 g(x)\,dx$$



$$\int_0^3 5\,dx + \int_3^5 x+2\,dx$$

$$5x\Big]_0^3 + \frac{x^2}{2}+2x\Big]_3^5$$

$$(15-0) + \left(\frac{25}{2}+10\right) - \left(\frac{9}{2}+6\right)$$

⑥ $\int_0^5 1+2x^3\,dx$

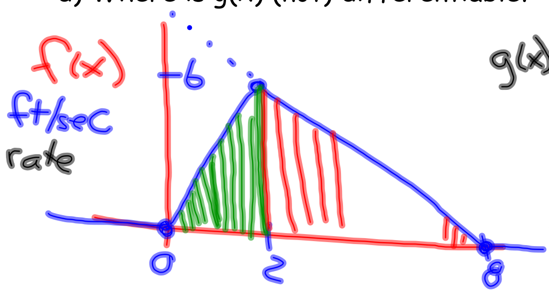
$\lim_{n\to\infty} \sum_{i=1}^{n} \left(\qquad \right)\left(\quad \right)$

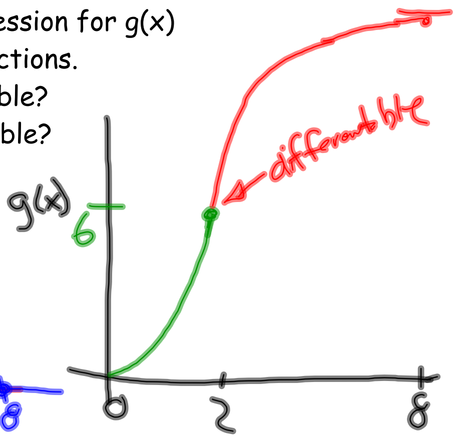
Similar to Homework Problem #73 from Sec 5.3

Let $f(x) = \begin{cases} 0 & \text{if} \quad x<0 \\ 3x & \text{if} \quad 0<x<2 \\ 8-x & \text{if} \quad 2\le x\le 8 \\ 0 & 8>x \end{cases}$ $\qquad g(x) = \int_0^x f(t)\,dt$

antiderivative

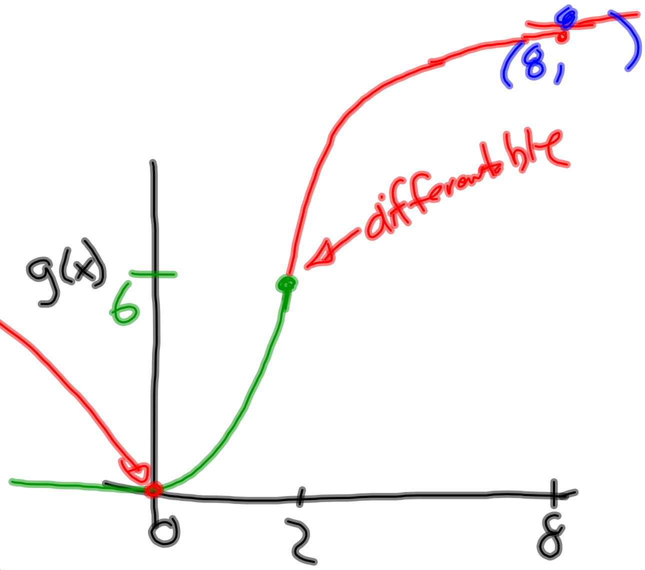
a) Find an piecewise defined expression for g(x)
b) Sketch the graphs of both functions.
c) Where is f(x) (not) differentiable?
d) Where is g(x) (not) differentiable?





Not differentiable at
x=2
x=0
x=8

$$g(x) = \begin{cases} 0 & x<0 \\ \frac{3}{2}x^2 + C \\ \frac{3}{2}x^2 + 0 & (\text{from } 0<x<2) \\ 8x - \frac{x^2}{2} + C & \text{from } (2<x<8) \\ 24 & x>8 \end{cases}$$



plug in (2,6)

$$6 = 8(2) - \frac{(2)^2}{2} + C$$

$$\boxed{C = -8}$$

Math 152 Calculus and Analytic Geometry II

Sec 5.4 Indefinite Integrals and the Net Change Theorem

We need a notation for antiderivatives now that we have the Fundamental Theorem Part I and II

The notation $\int f(x)dx$ is used for an antiderivative of f(x) and is called an <u>indefinite integral</u>

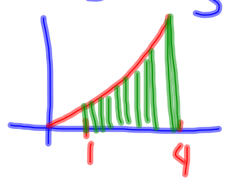
Notes:
The indefinite integral represents a family of functions (one for each +C)

The definite integral (with endpoints [a,b]) is a <u>number</u> (sum of rectangles)

$$\int x^2 dx = \frac{x^3}{3} + C$$

indefinite integral = family of functions

$$\int_1^4 x^2 dx = F(4) - F(1) = \frac{4^3}{3} - \frac{1^3}{3} = 21$$

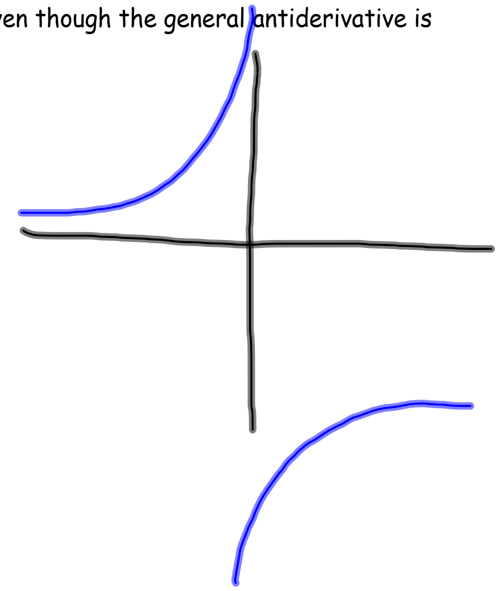


We adopt the convention that when a formula for a general indefinite integral is given, it is only valid on an interval.

For instance, we will write $\displaystyle\int \frac{1}{x^2}\,dx = -\frac{1}{x} + C$

even though the general antiderivative is

$$F(x) = \begin{cases} -\dfrac{1}{x} + C_1 & \text{if} \quad x < 0 \\ -\dfrac{1}{x} + C_2 & \text{if} \quad x \geq 0 \end{cases}$$



Try Some examples:

$$\int 4x - 5x^3 - 2\sqrt{x}\,dx$$

$$2x^2 - \frac{5x^4}{4} - \frac{2}{3}2\cdot x^{3/2} + C$$

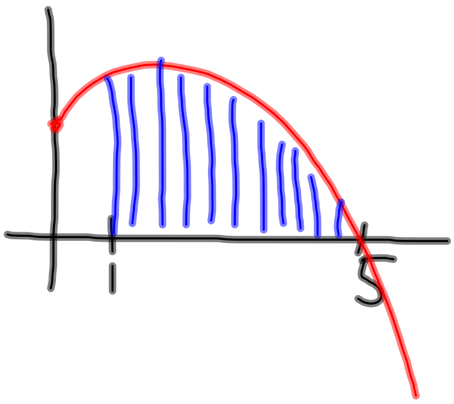
$$\int \left(\frac{4x - x^3}{x^2} \right) dx$$

$$\int \frac{4}{x} - x\,dx = 4\ln(x) - \frac{x^2}{2} + C$$

$$\int (\sec\theta \tan\theta - \cos\theta)\,d\theta = \sec(\theta) - \sin(\theta) + C$$

Evaluate the following definite integrals using Part II of FTC.

$$\int_1^5 7+3t-t^2\,dt = 7t + \frac{3t^2}{2} - \frac{t^3}{3}\Big]_1^5 = \left(7(5) + \frac{3(5)^2}{2} - \frac{(5)^3}{3}\right) - \left(7(1) + \frac{3(1)^2}{2} - \frac{(1)^3}{3}\right) = ?$$



$$\int_1^4 \frac{7x-\sqrt{x}}{x^2}\,dx =$$

The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x)dx = F(b) - F(a)$$

If Q(t) is the quantity of items stored in a warehouse, Q'(t) is the rate of change of the quantity at time t.
Find the total net change of quantity over the period from t=4 to t=8.

$$\text{Net Change} = \int_4^8 Q'(t)\,dt$$

Suppose we know that $Q'(t) = 15 - 3t$ in units per day. Find the net change in stored items over the 4 day-period.

$$\int_4^8 15-3t\,dt = 15t - \frac{3t^2}{2}\Big]_4^8$$

$$= (120-96) - (60-24)$$

$$= 24 - 36$$

$$= -12 \text{ units.}$$

http://www.calculusapplets.com/accumulation.html

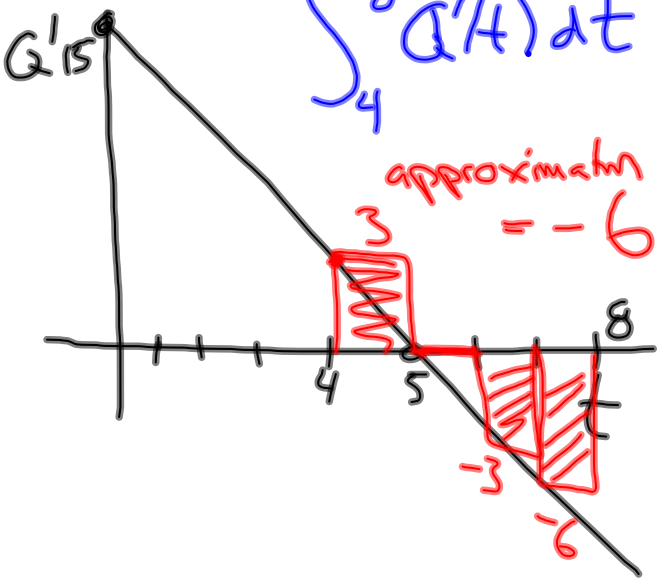
Suppose we know that $Q'(t) = 15 - 3t$ in units per day.

Estimate the net change in stored items over the 4-day period from t=4 to t=8

t	$Q'(t)$
0	15
1	12
2	9
3	6
4	3
5	0
6	-3
7	-6
8	-9

$$\int_4^8 Q'(t)\,dt$$

approximation = -6 units



The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x)dx = F(b) - F(a)$$

If P(t) is the population of a certain city, P'(t) is the rate of change of the population per year at time t. Find the total net change of quantity over the period from t=5 to t=10.

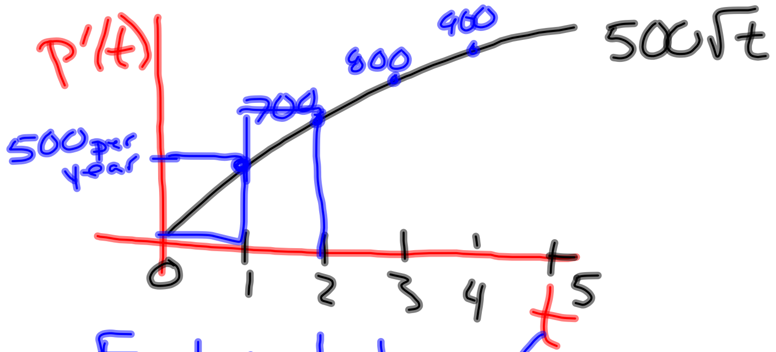
Net Change in Population $= \int_5^{10} P'(t) \cdot dt$

Suppose we know that $P'(t) = 500\sqrt{t}$ in people per year.

Find the net population change over the 5 year period.

$$P'(t) = 500\sqrt{t}$$

Change in Population from 5 to 10



Each rectangle = (500 people/year)·(1 year)

= 500 people

Total Area = Net Change in # of People

The Net Change Theorem: The definite integral of a rate of change is the "net change"

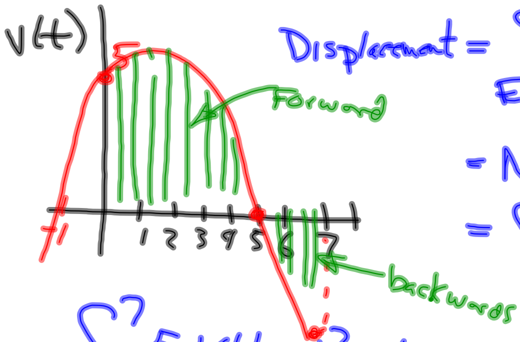
$$\int_a^b F'(x)dx = F(b) - F(a)$$

A particle moves along a straight path so that the velocity at time t is given by

$v(t) = 5 + 4t - t^2$ measured in feet per second.

$= (5-t)(1+t)$

Draw the velocity graph. Find the displacement of the particle from t=0 to t=7.



Displacement = $S(7) - S(0)$

Endpoint − beginning point

= Net Change in position

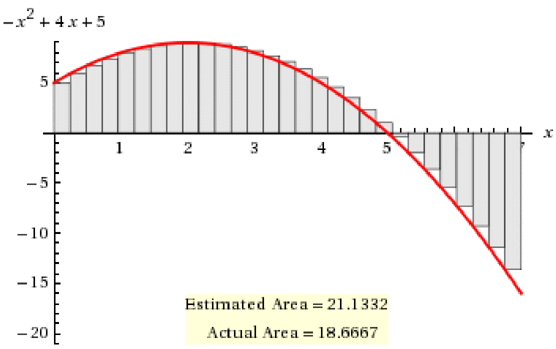
$= \int_0^7 v(t)\,dt$

$$\int_0^7 5 + 4t - t^2\,dt$$

$$\underbrace{5t + 2t^2 - \frac{t^3}{3}}_{\text{Position function.}}\Bigg]_0^7 = 35 + 98 - \frac{343}{3}$$

$= 18.666$

EXPLORE THIS TOPIC IN
The *MathWorld* Classroom



Graph the Riemann sum of: 5+4x-x^2

as x goes from 0 to 7

using 29 rectangles

taking samples at the Left

Print estimated and actual areas? ☑

Rectangle Color: Light Gray

Plot Color: Red

Replot

POWERED BY webMathematica

A particle moves along a straight path so that the velocity at time t is given by

$v(t) = 5 + 4t - t^2$ measured in feet per second.

Find the total distance traveled by the particle.

Not Displacement

mult. by -1 to make positive distance

$$\text{Distance} = \int_0^5 v(t)\,dt - \int_5^7 v(t)\,dt$$

$$= 5t + 2t^2 - \frac{t^3}{3}\Big]_0^5 - \left(5t + 2t^2 - \frac{t^3}{3}\right)\Big]_5^7$$

$$33.33 - (-14.666)$$

http://www.calculusapplets.com/eqofmotion.html

$= 48$ total

particle moves at

$v(t) = t^2 - t - 6$ ft/sec

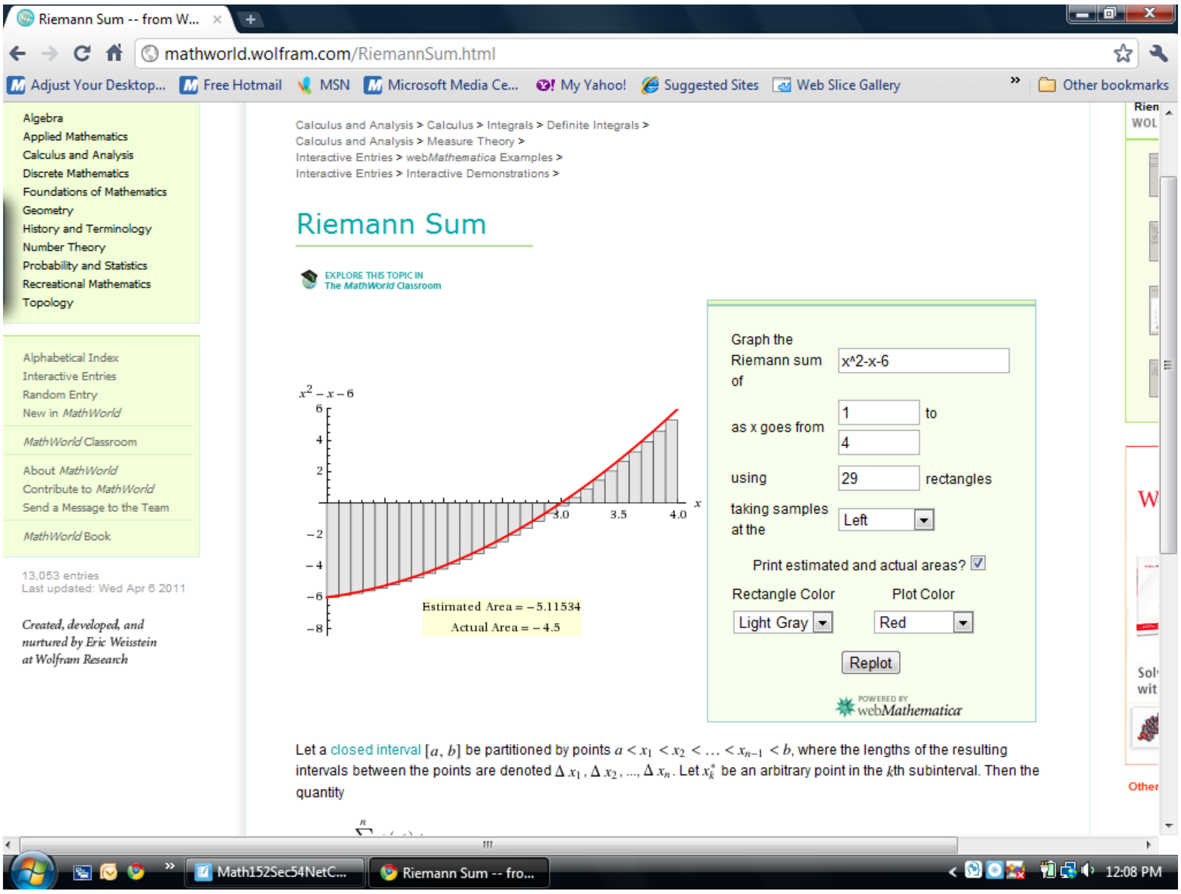
Find Distance & Displacement from $t = 1$ to $t = 4$

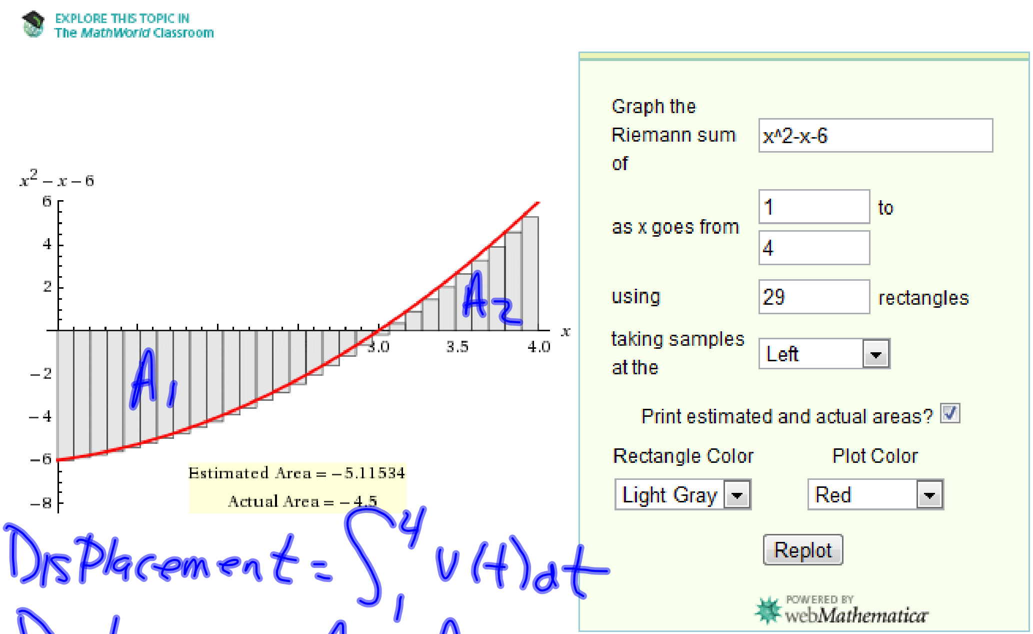
Displacement $\int_1^4 t^2 - t - 6\,dt$

$S(4) - S(0)$

$$= \frac{t^3}{3} - \frac{t^2}{2} - 6t\Big]_1^4 =$$

http://mathworld.wolfram.com/RiemannSum.html





Displacement $= \int_1^4 v(t)\,dt$

Distance $= A_1 + A_2$

Distance $= \int_1^4 |v(t)|\,dt$