

Quiz from Thursday

$$\int_0^5 x^2 + 3x \, dx$$

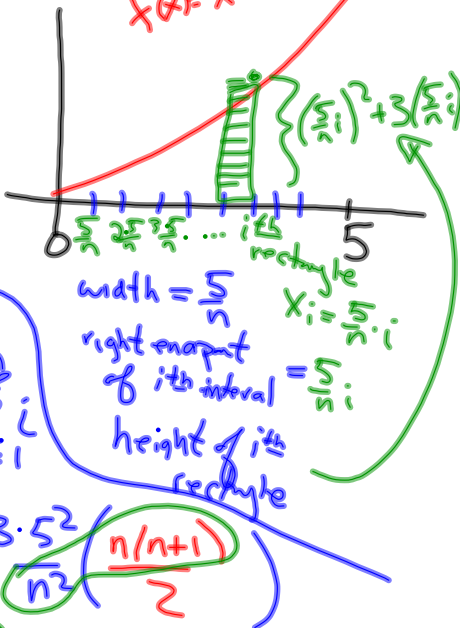
$$= \lim_{n \rightarrow \infty} \sum_{i=1}^n \left[\left(\frac{5}{n}i \right)^2 + 3 \left(\frac{5}{n}i \right) \right] \cdot \left(\frac{5}{n} \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{5^3}{n^3} \sum_{i=1}^n i^2 + 3 \left(\frac{5}{n} \right)^2 \sum_{i=1}^n i \right)$$

$$= \lim_{n \rightarrow \infty} \left(\frac{5^3}{n^3} \left(\frac{n(n+1)(2n+1)}{6} \right) + 3 \cdot \frac{5^2}{n^2} \left(\frac{n(n+1)}{2} \right) \right)$$

$$= \frac{5^3 \cdot 2}{6} + \frac{3 \cdot 5^2}{2}$$

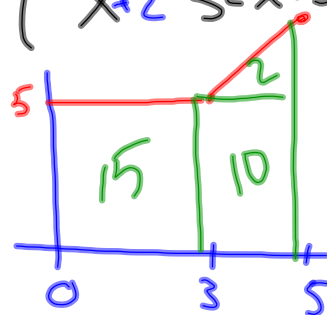
$$f(x) = x^2 + 3x$$



$$\int_0^5 g(x) \, dx \quad g(x) = \begin{cases} 5 & 0 \leq x < 3 \\ x+2 & 3 \leq x \leq 5 \end{cases}$$

$$\int_0^3 g(x) \, dx + \int_3^5 g(x) \, dx$$

$$\int_0^3 5 \, dx + \int_3^5 x+2 \, dx$$



$$\left[5x \right]_0^3 + \left[\frac{x^2}{2} + 2x \right]_3^5$$

$$(15 - 0) + \left(\frac{25}{2} + 10 \right) - \left(\frac{9}{2} + 6 \right)$$

⑥ $\int_0^5 1 + 2x^3 dx$

$\lim_{n \rightarrow \infty} \sum_{i=1}^n ($

)



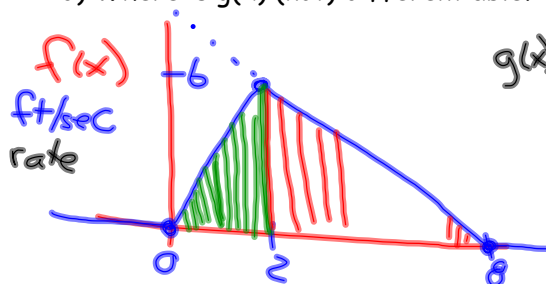
Similar to Homework Problem #73 from Sec 5.3

Let $f(x) = \begin{cases} 0 & \text{if } x < 0 \\ 3x & \text{if } 0 < x < 2 \\ 8-x & \text{if } 2 \leq x \leq 8 \\ 0 & \text{if } x > 8 \end{cases}$

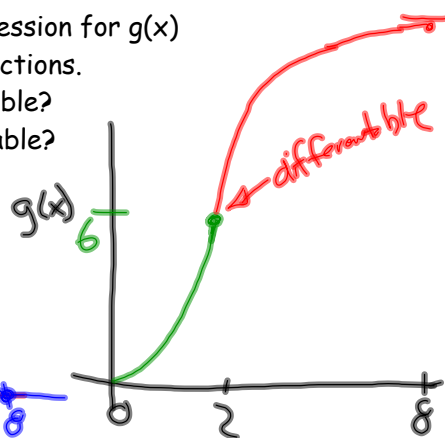
$g(x) = \int_0^x f(t) dt$

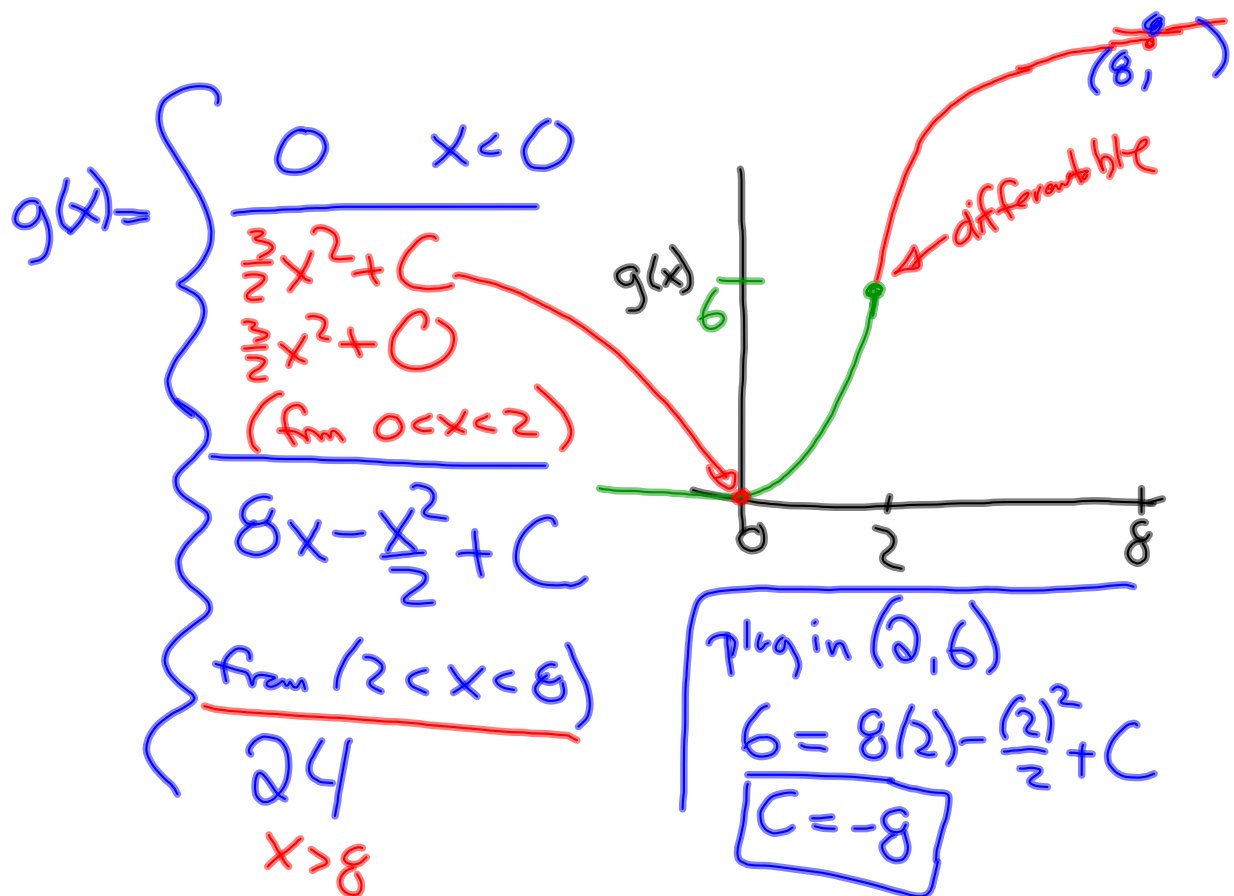
antiderivative

- Find a piecewise defined expression for $g(x)$
- Sketch the graphs of both functions.
- Where is $f(x)$ (not) differentiable?
- Where is $g(x)$ (not) differentiable?



Not differentiable at
 $x=2$
 $x=0$
 $x=8$





Math 152 Calculus and Analytic Geometry II

Sec 5.4 Indefinite Integrals and the Net Change Theorem

We need a notation for antiderivatives now that we have the Fundamental Theorem Part I and II

The notation $\int f(x) dx$ is used for an antiderivative of $f(x)$ and is called an indefinite integral

Notes:

The indefinite integral represents a family of functions (one for each $+C$)

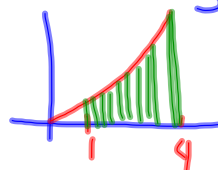
The definite integral (with endpoints $[a, b]$) is a number (sum of rectangles)

$$\int x^2 dx = \frac{x^3}{3} + C$$

indefinite integral
 = family of functions

$$\int_1^4 x^2 dx = F(4) - F(1)$$

$$= \frac{4^3}{3} - \frac{1^3}{3} = 21$$

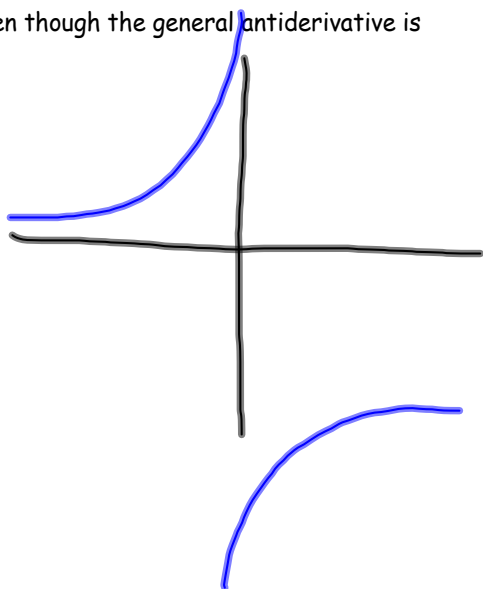


We adopt the convention that when a formula for a general indefinite integral is given, it is only valid on an interval.

For instance, we will write $\int \frac{1}{x^2} dx = -\frac{1}{x} + C$

even though the general antiderivative is

$$F(x) = \begin{cases} -\frac{1}{x} + C_1 & \text{if } x < 0 \\ -\frac{1}{x} + C_2 & \text{if } x \geq 0 \end{cases}$$



Try Some examples:

$$\int 4x - 5x^3 - 2\sqrt{x} dx$$

$$2x^2 - \frac{5x^4}{4} - \frac{2}{\frac{3}{2}} \cdot x^{\frac{3}{2}} + C$$

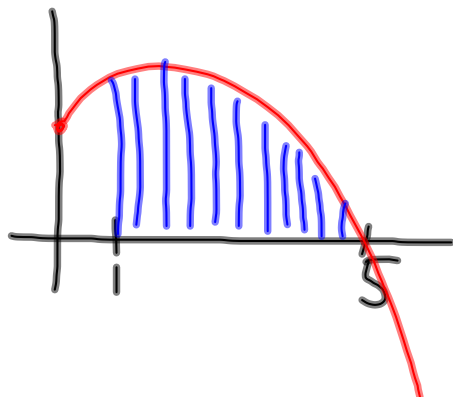
$$\int \left(\frac{4x - x^3}{x^2} \right) dx$$

$$\int \frac{4}{x} - x dx = 4 \ln(x) - \frac{x^2}{2} + C$$

$$\int (\sec \theta \tan \theta - \cos \theta) d\theta = \sec(\theta) - \sin(\theta) + C$$

Evaluate the following definite integrals using Part II of FTC.

$$\int_1^5 7 + 3t - t^2 dt = \left[7t + \frac{3t^2}{2} - \frac{t^3}{3} \right]_1^5 = \left(7(5) + \frac{3(5)^2}{2} - \frac{(5)^3}{3} \right) - \left(7(1) + \frac{3(1)^2}{2} - \frac{(1)^3}{3} \right) = ?$$



$$\int_1^4 \frac{7x - \sqrt{x}}{x^2} dx =$$

The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x) dx = F(b) - F(a)$$

If $Q(t)$ is the quantity of items stored in a warehouse, $Q'(t)$ is the rate of change of the quantity at time t .

Find the total net change of quantity over the period from $t=4$ to $t=8$.

$$\text{Net Change} = \int_4^8 Q'(t) dt$$

Suppose we know that $Q'(t) = 15 - 3t$ in units per day.

Find the net change in stored items over the 4 day-period.

$$\int_4^8 15 - 3t dt = \left[15t - \frac{3t^2}{2} \right]_4^8$$

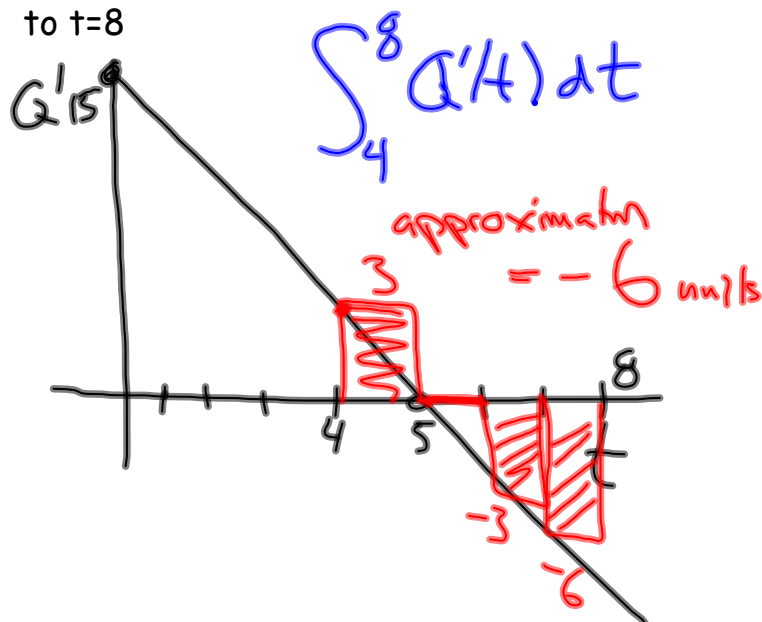
$$= (120 - 96) - (60 - 24) = 24 - 36 = -12 \text{ units.}$$

<http://www.calculusapplets.com/accumulation.html>

Suppose we know that $Q'(t) = 15 - 3t$ in units per day.

t	$Q'(t)$
0	15
1	12
2	9
3	6
4	3
5	0
6	-3
7	-6
8	-9

Estimate the net change in stored items over the 4-day period from $t=4$ to $t=8$



The Net Change Theorem: The definite integral of a rate of change is the "net change"

$$\int_a^b F'(x) dx = F(b) - F(a)$$

If $P(t)$ is the population of a certain city, $P'(t)$ is the rate of change of the population per year at time t . Find the total net change of quantity over the period from $t=5$ to $t=10$.

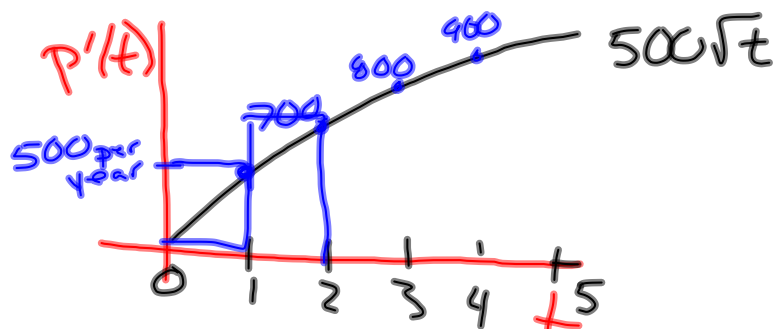
Net Change in Population = $\int_5^{10} P'(t) \cdot dt$

Suppose we know that $P'(t) = 500\sqrt{t}$ in people per year.

Find the net population change over the 5 year period.

$$P'(t) = 500\sqrt{t}$$

Change in Populatin from 5 to 10



$$\text{Each rectnyle} = (500 \text{ people/year}) \cdot (1 \text{ year})$$

$$= 500 \text{ people}$$

Total Area = Net Change in # of People

The Net Change Theorem: The definite integral of a rate of change is the "net change"

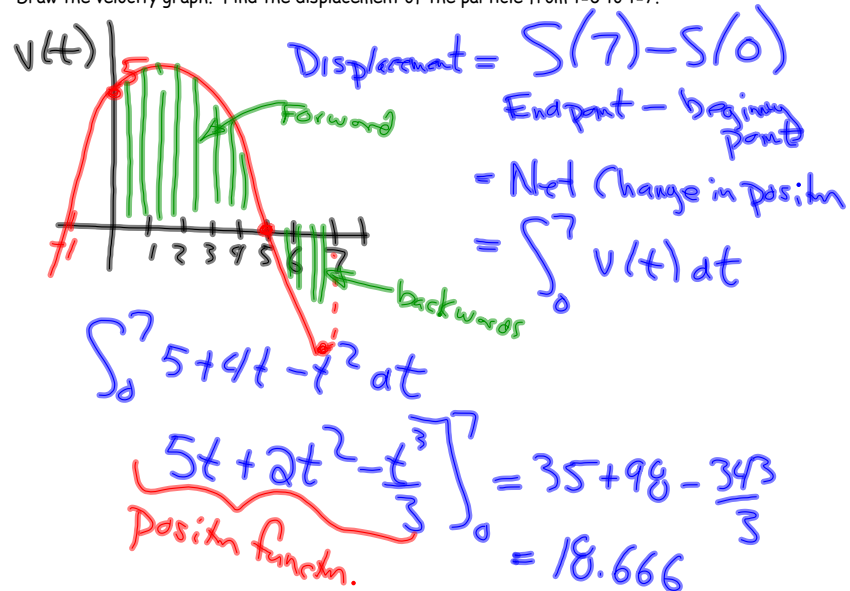
$$\int_a^b F'(x) dx = F(b) - F(a)$$

A particle moves along a straight path so that the velocity at time t is given by

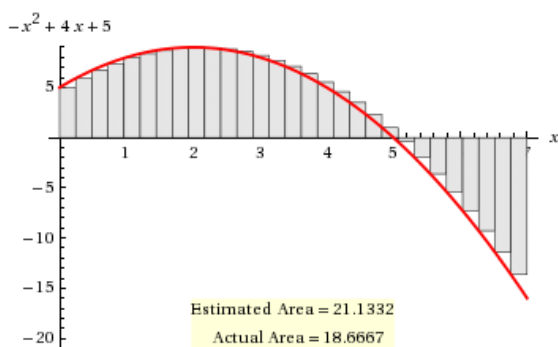
$$v(t) = 5 + 4t - t^2 \text{ measured in feet per second.}$$

$$= (5-t)(1+t)$$

Draw the velocity graph. Find the displacement of the particle from $t=0$ to $t=7$.



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Graph the
 Riemann sum of
 as x goes from to
 using rectangles
 taking samples at the

Print estimated and actual areas? ☒

Rectangle Color Plot Color

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A particle moves along a straight path so that the velocity at time t is given by

$$v(t) = 5 + 4t - t^2 \text{ measured in feet per second.}$$

Find the total distance traveled by the particle.

Not Displacement

$$\begin{aligned} \text{Distance} &= \int_0^5 v(t) dt - \int_5^7 v(t) dt \\ &= \left[5t + 2t^2 - \frac{t^3}{3} \right]_0^5 - \left[5t + 2t^2 - \frac{t^3}{3} \right]_5^7 \\ &= 33.33 - (-14.666) \\ &= 48 \text{ total} \end{aligned}$$

mult. by -1
to make positive
distance

<http://www.calculusapplets.com/eqofmotion.html>

particle moves at

$$v(t) = t^2 - t - 6 \text{ ft/sec}$$

Find Distance + Displacement from $t=1$ to $t=4$

$$\begin{aligned} \text{Displacement} &= \int_1^4 (t^2 - t - 6) dt \\ &= \left[\frac{t^3}{3} - \frac{t^2}{2} - 6t \right]_1^4 = \end{aligned}$$

<http://mathworld.wolfram.com/RiemannSum.html>

Riemann Sum -- from W...

mathworld.wolfram.com/RiemannSum.html

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Riemann Sum

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Graph the Riemann sum of $x^2 - x - 6$ as x goes from 1 to 4 using 29 rectangles taking samples at the Left.

Print estimated and actual areas? ☒

Rectangle Color: Light Gray Plot Color: Red

Replot

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Let a closed interval $[a, b]$ be partitioned by points $a < x_1 < x_2 < \dots < x_{n-1} < b$, where the lengths of the resulting intervals between the points are denoted $\Delta x_1, \Delta x_2, \dots, \Delta x_n$. Let x_k^* be an arbitrary point in the k th subinterval. Then the quantity

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Displacement = $\int_1^4 v(t) dt$
Distance = $A_1 + A_2$
Distance = $\int_1^4 |v(x)| dt$