HW 5.4

#8

$$\int_0^{\pi/3} \frac{\sin(\theta)+\sin(\theta)\tan^2(\theta)}{\sec^2(\theta)}\,d\theta$$

Fill in the missing steps.....

$$\int_0^{\pi/3} \sin(\theta)\,d\theta$$

4. $\frac{1}{3}\int 3t^2(t^3+4)^{-1/2}\,dt$

substitute $\Longrightarrow$ $\frac{1}{3}\int (u)^{-1/2}\,du$

$u = t^3+4$

$du = 3t^2\,dt$

$\frac{du}{3t^2} = dt$

$\frac{1}{3}\cdot\frac{2}{1}u^{1/2}+C$

$= \boxed{\frac{2}{3}(t^3+4)^{1/2}+C}$

12. $\int (x+1)\sin(x^2+2x+3)\,dx$

$u = x^2 + 2x + 3$

$du = 2x + 2\,dx = 2(x+1)dx$

$\frac{1}{2}\int \sin(x^2+2x+3)(2x+2)dx$

$= \frac{1}{2}\int \sin(u)\,du$

19. $\int \sqrt{x}\sqrt{(x\sqrt{x}+1)}\,dx$

$u = x\sqrt{x} + 1$

$u = x^{3/2} + 1$

$du = \frac{3}{2}x^{1/2}dx$

$\frac{2}{3}\int \frac{3}{2}\sqrt{x}(x^{3/2}+1)^{1/2}dx$

$\frac{2}{3}\int u^{1/2}du = \frac{2}{3}\frac{2}{3}u^{3/2} + C$

$\boxed{= \frac{4}{9}(x^{3/2}+1)^{3/2} + C}$

26. $\frac{1}{2}\int 2x \tan(x^2)\sec(x^2)\,dx$

$u = x^2$

$du = 2x\,dx$

$$\frac{1}{2}\int \tan(u)\sec(u)\,du$$

$$= \frac{1}{2}\sec(u) + C$$

$$= \frac{1}{2}\sec(x^2) + C$$

$$\frac{1}{8}\int \frac{8\,5x\sin\left(\sqrt{3+4x^2}\right)}{\sqrt{3+4x^2}}\,dx$$

$u = 3+4x^2$

$du = 8x\,dx$

or

$u = \sqrt{3+4x^2}$

$du = \frac{1}{2}\frac{1}{\sqrt{3+4x^2}}\cdot 8x\,d$

This works too but maybe harder to se

$$\frac{2}{1}\frac{5}{8}\int \frac{1}{2}\frac{\sin(\sqrt{u})}{\sqrt{u}}\,du \Rightarrow dv$$

2nd substitution

$V = \sqrt{u}$

$dv = \frac{1}{2}u^{-1/2}\,du$

$$\frac{5}{4}\int \sin(v)\,dv$$

$$-\frac{5}{4}\cos(v) + C$$

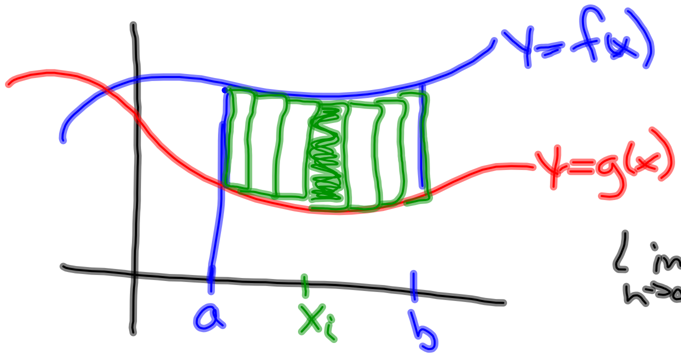
$$-\frac{5}{4}\cos(\sqrt{u}) + C$$

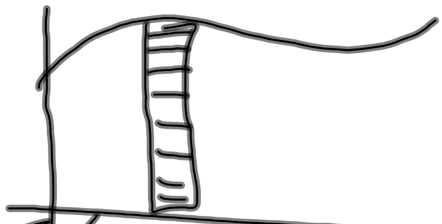
$$\boxed{-\frac{5}{4}\cos\left(\sqrt{3+4x^2}\right) + C}$$

Math 152 Calculus and Analytic Geometry II

Sec 6.1 Areas between curves

We can use definite integrals to find areas between two curves as follows:





$$\lim_{n\to\infty} \sum (f(x_i)-0)\Delta x$$

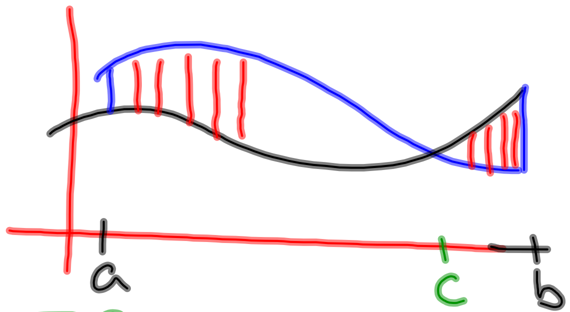
height of rectangle

$$Area = \lim_{n\to\infty}\sum_{i=1}^{n}(height)\Delta x \qquad \lim_{n\to\infty}\sum_{i=1}^{n}\left(f(x_i)-g(x_i)\right)\Delta x$$

The Area A of the region bounded by the curves y=f(x) and y=g(x) and the lines x=a, x=b, where f(x) and g(x) are continuous and f(x)>g(x) for all x in [a,b], is

Top − Bottom

$$A = \int_a^b \left(f(x_i)-g(x_i)\right)dx$$

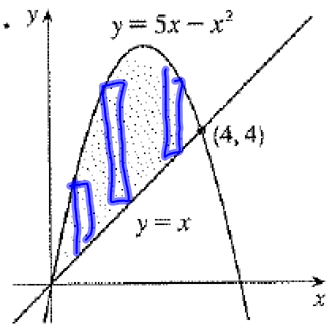


If they cross.

$$Area = \int_a^c (f(x)-g(x))dx + \int_c^b (g(x)-f(x))dx$$

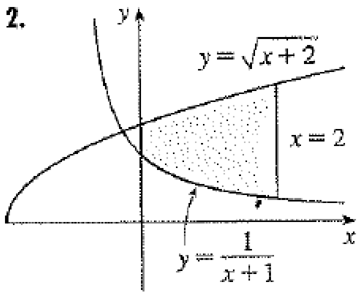
Examples:

1.



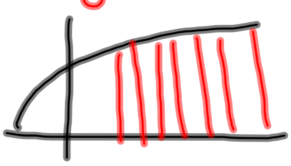
$$\text{Area} = \int_0^4 (5x - x^2) - x \quad dx$$

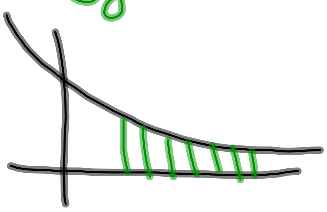
2.



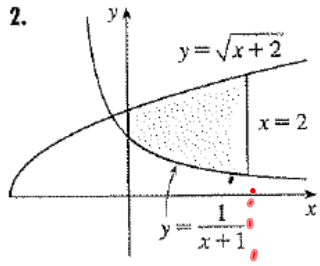
$$\text{Area} = \int_0^2 \left(\sqrt{x+2} - \frac{1}{x+1}\right) dx$$

$$\int_0^2 \sqrt{x+2}\, dx - \int_0^2 \frac{1}{x+1}\, dx$$

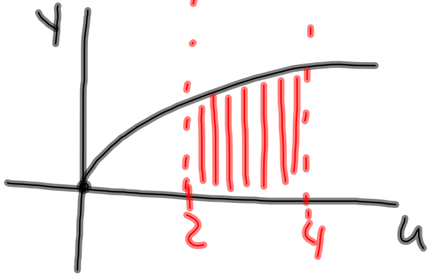




$$\int_0^2 \sqrt{x+2}\,dx$$



Substitute $u = x+2$



$u = x+2$

$du = dx$

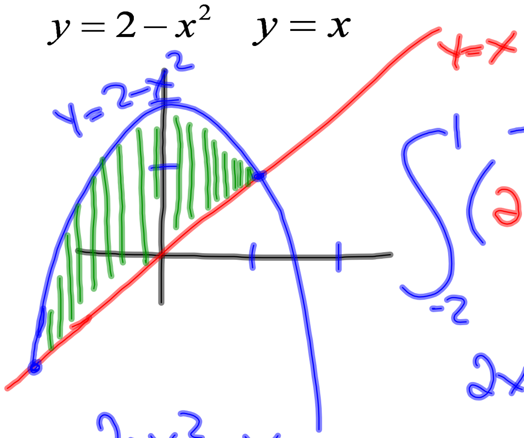
$$\int_2^4 \sqrt{u}\,du$$

$$= \frac{2}{3}u^{3/2}\Big]_2^4$$

$$= \boxed{\frac{2}{3}\left(4^{3/2}-2^{3/2}\right)}$$

Find the area of the region bounded by:

$y = 2 - x^2 \quad y = x$



$$\int_{-2}^{1} \overbrace{(2-x^2)}^{\text{Top}} - \overbrace{(x)}^{\text{Bottom}}\,dx$$

$$2x - \frac{x^3}{3} - \frac{x^2}{2}\Big]_{-2}^{1}$$

$$= \left(2-\frac{1}{3}-\frac{1}{2}\right) - \left(-4-\left(\frac{-8}{3}\right)-\frac{4}{2}\right)$$

$2 - x^2 = x$

$0 = x^2 + x - 2$

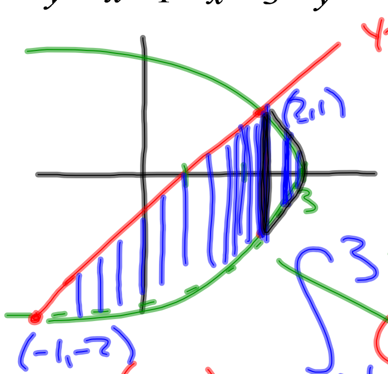
$0 = (x+2)(x-1)$

$x = -2$

$x = 1$

Find the area of the region bounded by:

$y = x - 1 \quad x = 3 - y^2$



$y^2 = 3 - x$

$y = \pm\sqrt{3-x}$

$\cancel{\int_{-1}^{3} \overset{\text{Top}}{(x-1)} - \overset{\text{Bottom}}{(-\sqrt{3-x})}}$

$\int_{-1}^{2} \quad + \int_{2}^{3}$

$y = (3-y^2) - 1$

$y^2 + y - 2 = 0$

$(y+2)(y-1) = 0$

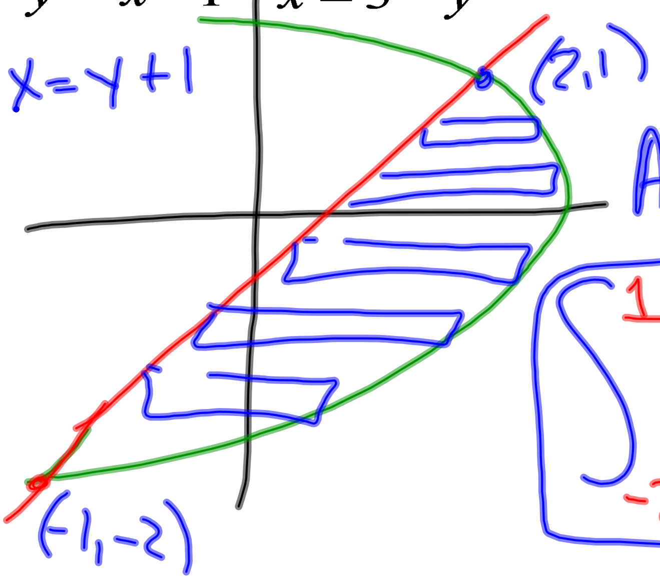
$y = -2 \quad y = 1$

$x = -1 \quad x = 2$

$$\int_{-1}^{2} (x-1) - (-\sqrt{3-x})\,dx + \int_{2}^{3} (\sqrt{3-x}) - (-\sqrt{3-x})\,dx$$

$y = x - 1 \quad | \quad x = 3 - y^2$

$x = y + 1$



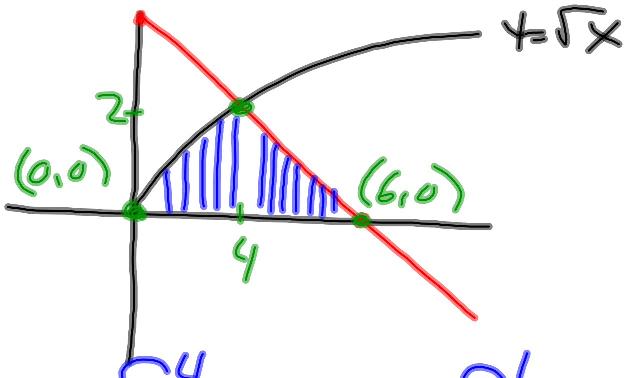
with respect to y

$\text{Area} = \int (\text{Right}) - (\text{Left})\,dy$

$$\boxed{\int_{-2}^{1} (3-y^2) - (y+1)\,dy}$$

Find the area of the region bounded by:

$y=\sqrt{x}$ $\quad y=6-x$ $\quad$ and the x-axis



$6-x=\sqrt{x}$

$x-\sqrt{x}-6=0$

$w^2-w-6=0$

$(w+3)(w-2)=0$

$(\sqrt{x}+3)(\sqrt{x}-2)=0$

$\sqrt{x}=-3 \qquad \sqrt{x}=2$

$x=9 \qquad x=4$

$$\text{Area}=\int_0^4(\sqrt{x}-0)\,dx+\int_4^6(6-x-0)\,dx$$

Find the area of the region bounded by:

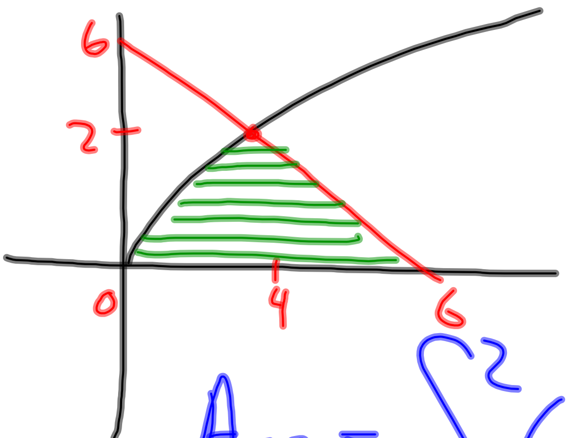
$y=\sqrt{x}$ $\quad y=6-x$ $\quad$ and the x-axis

$y^2=x$

Do it again with horizontal rectangles

$y=6-x$

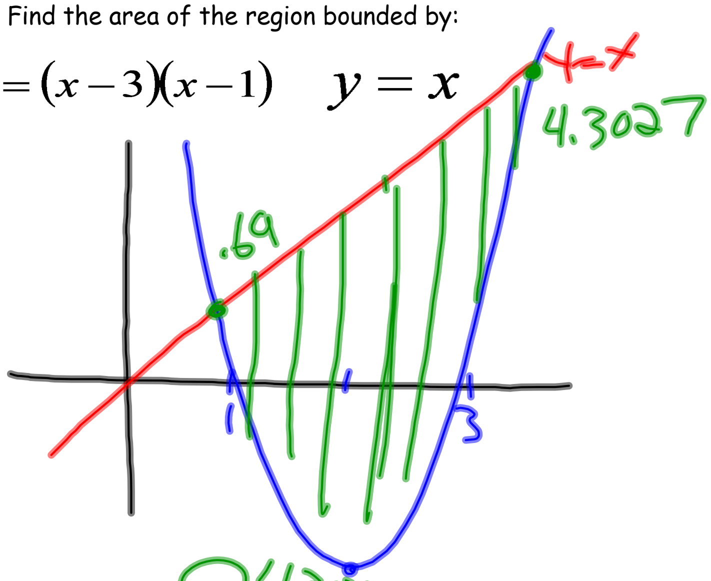
$x=6-y$



$$\text{Area}=\int_0^2 \overset{\text{right}}{(6-y)}-\overset{\text{left}}{(y^2)}\,dy$$

Find the area of the region bounded by:

$y=(x-3)(x-1) \quad y=x$



$$\int_{.69}^{4.3027} x-((x-3)(x-1))\,dx$$

Review Problems from Chapter 5 Review (Page 431)

Try the following problems: 2(a,b,c,d), 4, 7, 8, 9-31 odd , 43, 45, 47

Harder... 66, 67, 68, 69, 70