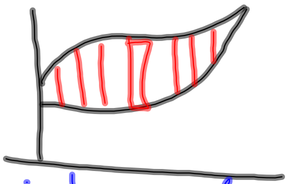
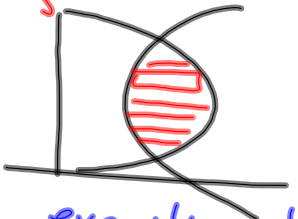
Disks/washers vs Shells

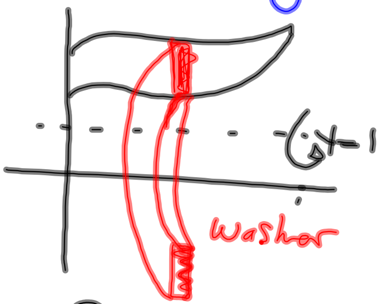
1. Pick Horizontal or Vertical Rectangles



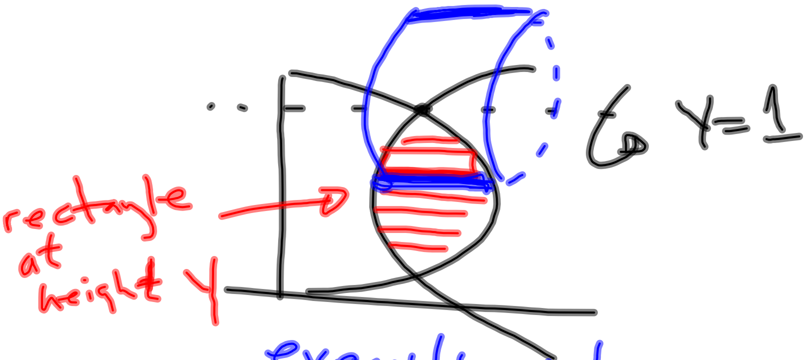
in terms of x



everything in terms of y



$$V = \int \pi(\text{outside} - \text{axis})^2 - \pi(\text{inside} - \text{axis})^2$$

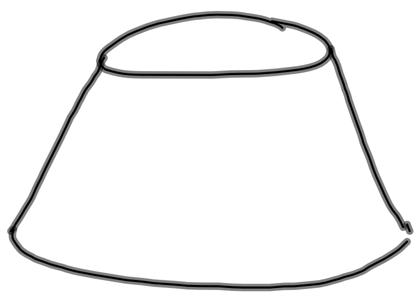


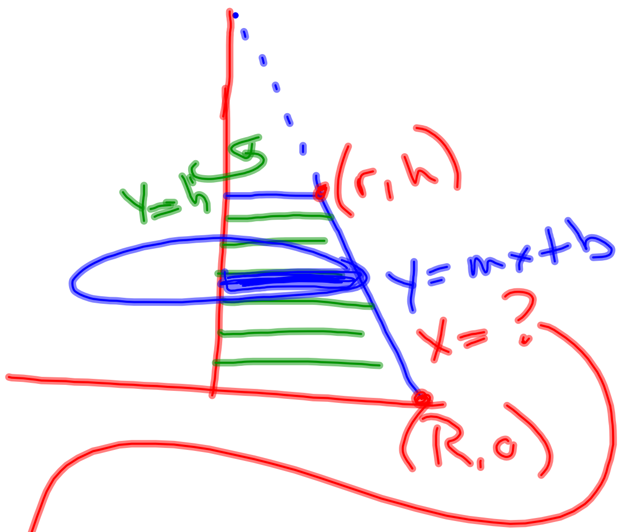
everything in terms of y

$$V = \int 2\pi \overbrace{(\text{axis} - y)}^{\text{radius}} \overbrace{(\text{right} - \text{left})}^{\text{height}}\, dy$$

$$(1 - y)$$

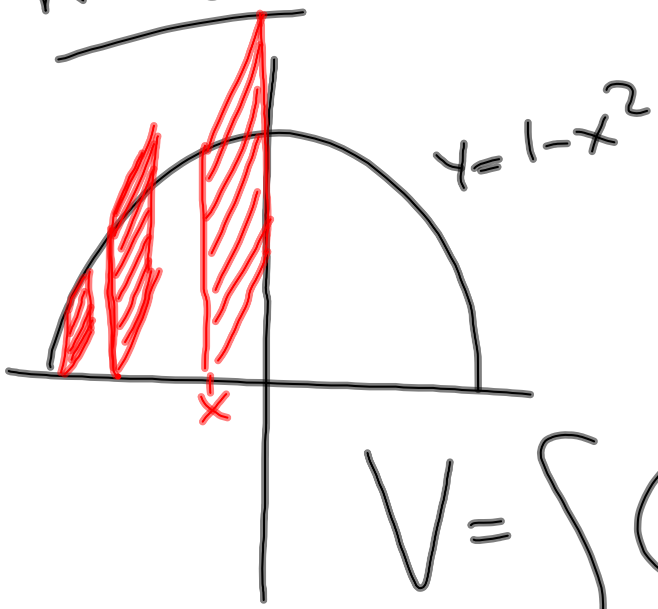
HW 6.2





$$V = \int_0^h \pi\left(r(\ ) - 0\right)^2 dy$$

HW 6.2



Area of cross section

$$V = \int \left((1-x^2)^2\right) \cdot dx$$

Math 152 Calculus and Analytic Geometry II

Sec 6.4 Work

Work = Force * Distance where Force = mass * acceleration

$W = F \cdot d$ $F = m \cdot a$

Example: How much work is required to lift a 10kg box to a height of 3 meters?

What is the force required?

$F = m \cdot a = (10\text{ kg})(9.8\text{ m/s}^2)$ Newtons (gravity)

$W = F \cdot d = (10\text{ kg})(9.8\text{ m/s}^2)(3\text{ meters})$

$= 294$ Joules

or Newton · Meters

Units for Force and Work (in metric and US Customary)

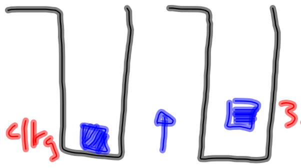
	Metric	US Customary
mass	Kg	Pounds? (slugs)
displacement	meters	feet
Time	seconds	seconds
Force	Newtons = Kg · m/s²	pounds
Work	Newton · Meters Force · (accel) · (distance)	foot · pounds

What if the Force required for the work is not constant?

Break the work up into short segments and approximate the work done on each segment. Then add them up.

Suppose you are pulling a bucket of water up a well on a pulley. But the water is leaking out at a rate of 0.5 kg per meter. Assume the well is 6 meters deep and the full bucket has a mass of 4kg.

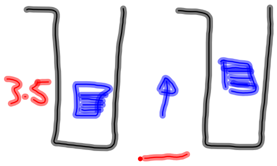
Approximately how much work is required to lift the bucket the first meter?



Use left endpoint to estimate $F = 4$ kg

$$W = F \cdot d = (4\text{ kg})(9.8\text{ m/s}^2)(1\text{ meter})$$
$$= 39.2 \text{ Newtons}$$

Approximately how much work is required to lift the bucket the second meter?



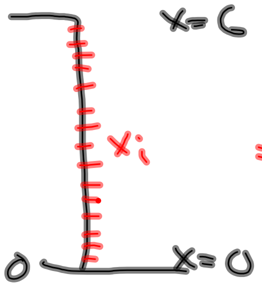
$$W = (3.5\text{ kg})(9.8\text{ m/s}^2)(1\text{ meter})$$
$$=$$

Can we find an integral to compute the total work needed to raise the leaky bucket to the top of the well?

$$F = \sum_{i=1}^{n} F(x_i) \cdot \Delta x$$

Force

Distance over each short interval



$$= \int_0^6 F(x) \cdot dx$$

$$= \int_0^6 \underbrace{(4 - \tfrac{1}{2}x)}_{\text{kg}} \underbrace{(9.8)}_{\text{m/s}^2} (dx \text{ meters})$$

$$= 9.8 \int_0^6 4 - \tfrac{1}{2}x \, dx$$

Work done by springs:

Hooke's Law: The force required to hold a spring stretched x units past its natural length is proportional to x

k = the spring constant.

$F = k \cdot x$

Newtons

A force of 40 N is required to hold a spring (naturally 10 cm long) at a length of 15 cm. How much force is required to stretch it from 15 to 18 cm?

How do you find "K"?

$F = k \cdot x$

$40\,N = k \cdot (5\,cm)$

Force needed   distance stretched

$$W = \int_5^8 F(x) \cdot dx = \int_5^8 k \cdot x \, dx$$

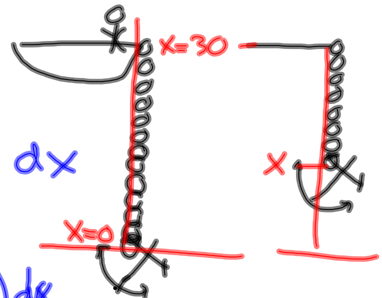
$$= \int_5^8 8x \, dx$$

http://archives.math.utk.edu/visual.calculus/5/work.1/index.html

Work to lift an anchor.

Suppose your ship has a 30 meter chain with an anchor that is just touching the ocean floor. It has a mass of 450 kg.

How much work is required to lift the chain to the surface? (Ignore the actual anchor, we can do it later if we want.)



$F = m \cdot a$

$$W = \int_0^{30} F \cdot dx$$

$$W = \int_0^{30} (\text{Mass})(9.8)\, dx$$

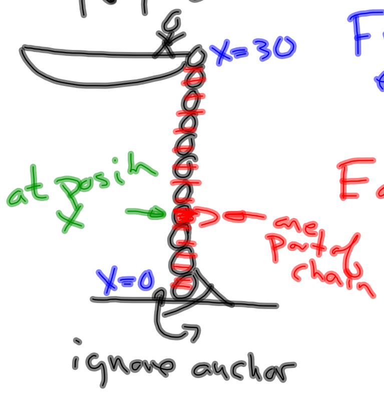
Chain $= \dfrac{450\,kg}{30\text{ meters}} = 15\,kg/m$

$$W = \int_0^{30} \big(\underbrace{(30-x)}_{\text{length}} \cdot \underbrace{15}_{(kg/meter)}\big)\, 9.8\, dx$$

with anchor

$$W = \int_0^{30} \Big(\underbrace{(30-x) \cdot 15}_{\text{mass of part of chain}} + \underbrace{\text{Anchor kg}}_{\text{mass of Anchor}}\Big)(9.8)\, dx$$

Try another idea



Find the work to raise each interval of chain separately then "add them up"

Each link is "dx" long

mass $= (15\ \mathrm{kg/m}) \cdot dx$ meters

$= 15\,dx$

Force $= m \cdot a$

$= (15 \cdot dx)(9.8)$

Work $= F \cdot d$

$= (15 \cdot dx)(9.8)(30-x)$

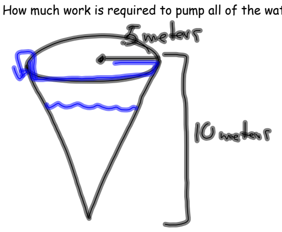
$$\text{Total work} = \int_0^{30} 15 \cdot (30-x)(9.8)\,dx$$

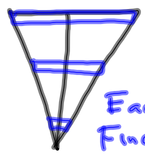
same as before

Work to empty a tank.

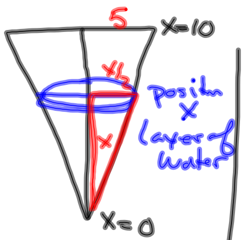
Suppose you have a tank of water in the shape of an inverted cone with height 10 meters and radius of the base 5 meters.

How much work is required to pump all of the water out the top of the tank?





Each layer
Find Mass
+ Distance to top.



Volume $= \pi\left(\frac{x}{2}\right)^2 dx$ Depth

Mass $=$ Volume $\cdot$ Density
$m^3 \cdot \mathrm{kg/m^3}$

$= \left(\pi\left(\frac{x}{2}\right)^2 \cdot dx\right)(1000)$

Force $=$ Mass $\cdot$ Acceleration
$1000\pi\left(\frac{x}{2}\right)^2 dx \cdot (9.8)$

Work $=$ Force $\cdot$ Distance
$= 1000\pi\left(\frac{x}{2}\right)^2 9.8\,(10-x)\,dx$

$$\boxed{\text{Total Work} = \int_0^{10} 9800\,\pi\left(\frac{x}{2}\right)^2 (10-x)\,dx}$$

How much force is required to move to electrons apart from a distance of ~~1~~ 4 picameter to ~~4~~ 1 picameters?

The force between two charges is proportional to the product of their charges and inversely proportional to the square of the distance between them.

An electron has a 1.6×10^{-19} C negative charge (in Coulombs). And a picometer is 10^{-12} meters. The proportionality constant $k = 9 \times 10^{9}$

Other applications of integration.

Mass of a region. Suppose the density of material over some region was given as a function D(x).

Mass = Area * D(x)

Slice with Vertical rectangles
Volume of each disk
Mass of each disk
$= V(x) \cdot D(x)$
$= A(x) \cdot D(x) \cdot dx$

$\int A(x) \cdot D(x)\, dx$

Center of Mass of a region

http://www.intmath.com/Applications-integration/5_Centroid-area.php

Moment of Inertia as a region rotates around an axis

http://www.intmath.com/Applications-integration/6_Moments-inertia.php