Math 152 Calculus and Analytic Geometry II

Sec. 7.1 Ingtegration By Parts

More techniques to find antiderivatives:

Chain Rule leads to Substitution

$$\frac{d}{dx}\left(f(u(x))\right) = f'(u(x)) \cdot u'(x)$$

$$f(u(x)) = \int \left(f'(u(x)) \cdot u'(x)\right) dx$$

Product Rule Leads to "Integration by Parts"

$$\frac{d}{dx}\left(u(x)v(x)\right) = u(x) \cdot v'(x) + u'(x)v(x)$$

"Integrate both sides"

Rearrange to get a useful identity.

$$\int u(x)v'(x)dx = u(x)v(x) - \int v(x)u'(x)dx$$

or simply

$$\int u \cdot dv = u \cdot v - \int v \cdot du$$

We will use this technique to switch from a difficult integral to one that is "hopefully" easier to evaluate.

We will have to recognize the original function as a product.

Pick one term to be 'u' and the other to be 'dv'

Example:

$$\int xSin(x)dx$$

Example:

$$\int x^2 x^3 dx$$

Example:

$$\int x^2 e^{-x} dx$$

Example:

$$\int \ln(x)dx$$

Example:

$$\int \tan^{-1}(x)dx$$

Find a "reduction" formula

$$\int x^n e^x dx$$

$$\int x^n e^x dx = x^n e^x - n \int x^{n-1} e^x dx$$

Evaluate

$$\int Sin^n(x) dx$$