Math 152 Calculus and Analytic Geometry II

Sec. 7.2 Trigonometric Integrals

This section has suggestions for how to solve integrals involving certain combinations of trig functions.

For example.

$\int Cos^3(x)dx$

rewrite it

$= \int \cos^2(x) \cdot \cos(x)\, dx$

$= \int (1 - \sin^2(x)) \underbrace{\cos(x) dx}_{=du}$

substitution

$u = \sin(x)$

$du = \cos(x)dx$

$= \int (1 - u^2)\, du$

$= u - \frac{u^3}{3} + C$

$= \boxed{\sin(x) - \frac{(\sin(x))^3}{3} + C}$

Check

$y' = \cos(x) - \frac{3(\sin(x))^2}{3} \cdot \cos(x)$

$\cos(x)(1 - \sin^2(x))$

For Example:

$$\int Cos^3(x)Sin^5(x)dx = \int (\cos(x))^2 \cdot (\sin x)^5 \cos(x)\,dx$$

$u = \sin(x)$

$du = \cos(x)dx$

$\frac{du}{\cos(x)} = dx$

$$= \int (1-\sin^2(x))(\sin(x))^5 \underbrace{\cos(x)dx}_{du}$$

$$\int (1-u^2)u^5\,du$$

$$= \int u^5 - u^7\,du$$

$$= \frac{u^6}{6} - \frac{u^8}{8} + C$$

$$= \frac{(\sin(x))^6}{6} - \frac{(\sin(x))^8}{8} + C$$

For Example:

$$\int Cos^3(x)Sin^5(x)dx$$

$u = \cos(x)$

$du = -\sin(x)dx$

$$= -\int (\cos(x))^3 ((\sin(x))^4)\underbrace{(-\sin(x)dx)}_{du}$$

$$(1-\cos^2(x))^2$$

$$= -\int u^3(1-u^2)^2 \cdot du$$

$$-\int u^3 - 2u^5 + u^7\,du$$

$$= -\frac{(\cos(x))^4}{4} + \frac{2(\cos(x))^6}{6} - \frac{(\cos(x))^8}{8} + C$$

These should be equal.

$$= \frac{(\sin(x))^6}{6} - \frac{(\sin(x))^8}{8} + C$$

What about this?

$$\int Sin^2(x)dx = \int \frac{1}{2}\left(1-\cos(2x)\right)dx$$

$$= \int \frac{1}{2} - \frac{1}{2}\cos(2x)\, dx$$

$$= \frac{1}{2}x - \frac{1}{2}\left(\frac{1}{2}\sin(2x)\right) + C$$

General Strategy:

$$\int Sin^{(odd)}(x)Cos^{(anything)}(x)dx = -\int (\sin(x))^{\text{even}}((\cos(x))^n\cdot\underbrace{(-\sin(x)dx)}_{du}$$

$(1-\cos^2(x))^m$

$u = \cos(x)$

$du = \sin(x)dx$

$$-\int (1-u^2)^m (u)^n du$$

$$\int Sin^{(anything)}(x)Cos^{(odd)}(x)dx = \int (\sin(x))^n (\cos(x)^2)^m (\cos(x)dx)$$

$u = \sin(x)$

$du = \cos(x)dx$

$$= \int u^n (1-u^2)^m \cdot du$$

What about

$$\int Tan^6(x)Sec^4(x)dx = \int (\tan(x))^6 \cdot \underbrace{\sec^2(x)}_{1+\tan^2(x)} \cdot \underbrace{\sec^2(x)dx}_{du}$$

$1+u^2$

$u = \tan(x)$

$du = \sec^2(x)dx$

$$= \int u^6 (1+u^2)\cdot du$$

Works if sec(x) to even power.

Another one...

$$\int Tan^7(x)Sec^4(x)dx$$

$u = sec(x)$

$du = sec(x)tan(x)dx$

$$= \int (tan(x))^6 \cdot \underbrace{(sec(x))^3}_{u^3} \cdot \underbrace{sec(x)tan(x)dx}_{du}$$

$(tan^2(x))^3$

$(sec^2 x - 1)^3$

$(u^2 - 1)^3$

$$\boxed{\int (u^2-1)^3 \cdot u^3 \cdot du}$$

This works when "tan" is to odd power.

General Strategy:

$$\int Tan^{(anything)}(x)Sec^{(even)}(x)dx$$

$u = tan(x)$

$du = sec^2(x)dx$

$$\int Tan^{(odd)}(x)Sec^{(anything)}(x)dx$$

$u = sec(x)$

$du = sec(x)tan(x)dx$

Otherwise, harder, maybe try some identities...

$$\int Tan(x)dx = \ln|Sec(x)| \qquad \int Sec(x)dx = \ln|Sec(x)+Tan(x)|$$

$$\int \frac{sin(x)}{cos(x)}dx$$

$u = cos(x)$

$$\int (\sin(x))^6\,dx$$

$$\int \left(\tfrac{1}{2}(1-\cos(2x))\right)^3 dx$$

really long....

HW Problems:

15. $\int \sin^3(x)\sqrt{\cos(x)}\,dx = -\int \sin^2(x)\sqrt{\cos(x)}\,(-\sin(x)\,dx)$

$u = \cos(x)$

$du = -\sin(x)\,dx$

$$= -\int (1-\cos^2(x))\sqrt{\cos(x)}\cdot(-\sin(x)\,dx)$$

$$= -\int (1-u^2)\sqrt{u}\,du$$

$$= -\int u^{1/2} - u^{5/2}\,du$$

$$= -\frac{2}{3}u^{3/2} + \frac{2}{7}u^{7/2} + C$$

$$= -\frac{2}{3}(\cos(x))^{3/2} + \frac{2}{7}(\cos(x))^{7/2} + C$$

31. $\int \frac{\tan^5(x)}{\cos^4(x)} dx$

$= \int (\tan(x))^5 (\sec(x))^4 \cdot dx$

$u = \tan(x)$
$du = \sec^2(x)$ works

$u = \sec(x)$
$du = \sec(x)\tan(x)dx$ works too

$$= \int \frac{\left(\frac{\sin(x)}{\cos(x)}\right)^5}{(\cos(x))^4} dx$$

$$= \int (\sin(x))^5 \cdot (\cos(x))^{-9} dx$$

$u = \sin(x)$
$du = \cos(x)dx$

$$= \int (\sin(x))^5 \cdot \frac{1}{(\cos(x))^{10}} \cdot \cos(x) dx$$

$$\int u^5 \cdot \frac{1}{(1-u^2)^5} \cdot du$$

This gets hard.

$u = \cos(x)$
$du = -\sin(x)dx$

$$= -\int (1-u^2)^2 \cdot u^{-9} \cdot du$$

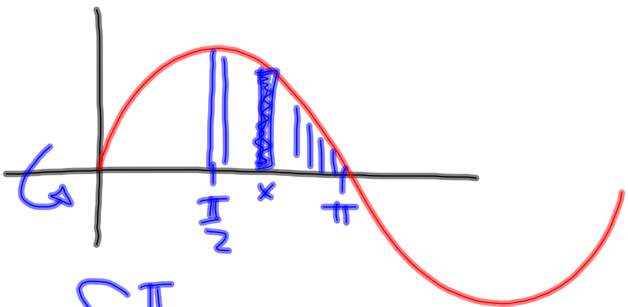
59. Rotate the given region about the x-axis

$y = \sin(x)$

$x = \frac{\pi}{2}$

$x = \pi$

$y = 0$



$$\text{Volume} = \int_{\pi/2}^{\pi} \pi(\sin(x))^2 dx$$

$$\int \pi(1-\cos^2(x))dx$$

$$= \pi \int_{\pi/2}^{\pi} \left(\tfrac{1}{2}(1-\cos(2x))\right)dx$$

$$\pi\left(\tfrac{1}{2}x - \tfrac{1}{4}\sin(2x)\right)\Big]_{\pi/2}^{\pi}$$