Math 152E Calculus and Analytic Geometry II

Sec. 7.3 Trigonometric Substitutions

This section has suggestions for how to solve integrals involving square roots by certain <u>substitutions</u> of trig functions.

For example.

$$\int_0^3 2\pi x \sqrt{9-x^2}\,dx$$

Compare that to the following: (An area problem, not a revolution)

$$\int_0^3 \sqrt{9-x^2}\,dx$$

Substitute:

$$x = 3Sin(\theta)$$

Regular Substitution

$$\int_{0}^{3} 2\pi x \sqrt{9 - x^2}\, dx$$

New variable is function of the old variable

Inverse Substitution

$$\int_{0}^{3} \sqrt{9 - x^2}\, dx$$

Old variable is function of new variable

Examples:

$$\int \frac{\sqrt{4 - x^2}}{x^2}\, dx$$

What about

$$\int \frac{x^3}{\sqrt{x^2+9}}\,dx$$

General Strategy....   Substitution: Identity to simplify

$$\sqrt{a^2-x^2}$$

$$\sqrt{a^2+x^2}$$

$$\sqrt{x^2-a^2}$$