Math 152 Calculus and Analytic Geometry II

Sec. 7.8 Improper Integrals and Sec. 4.4 L'Hopital's Rule

Find the Area under the curve y=f(x) and above y=0 and to the right of x=1.

$$y = \frac{1}{x^2}$$

Definition of Improper Integral I

If the integral from a to 't' exists for every 't', then

$$\int_a^\infty f(x)dx = \lim_{t \to \infty} \int_a^t f(x)dx$$

Similar for Limits to negative infinity...

The improper integral is <u>convergent</u> if

The improper integral is <u>divergent</u> if

Find the Area under the curve y=f(x) and above y=0 and to the right of x=1.

$y = 1/x$

Compare the Graphs of the two functions

Evaluate the following: (Graph it first)

$$\int_0^\infty xe^{-2x}dx$$

Sec. 4.4 L'Hopital's Rule

If Lim f(x)=0 and Lim g(x)=0 as x approaches 'a', then $$\lim_{x\to a}\frac{f(x)}{g(x)}=\lim_{x\to a}\frac{f'(x)}{g'(x)}$$

$$\lim_{x\to 1}\frac{x^2-5x+4}{x-1}$$

Indeterminate Forms: All can be rewritten into form of 0/0 to use L'Hopital

$$\lim_{t \to 0^+} t \ln(t)$$

Sec. 4.4 L'Hopital's Rule and Indeterminate Forms

Suppose you are evaluating a limit (a fraction, product or exponent) by "plugging in" or finding the limits of each part separatly. The limit will have one of the following forms. Each "part" can approach either zero, infinity or a constant.

Exponential: Possible "Forms"

$$\lim_{x \to a} f(x)^{g(x)}$$

Results or "Indeterminate"

$$\lim_{x \to \infty} (x^7 - 6x^4 + 3x - 7)e^{(-3x)}$$

Indeterminate Forms: All can be rewritten into form of 0/0 to use L'Hopital

$$\lim_{t\to\infty} te^{-2t}$$

$$\lim_{x\to 0} x^x$$

Back to Improper Integrals

Evaluate the following: (Graph it first)

$$\int_{-\infty}^{\infty} \frac{1}{\pi\left(1+x^2\right)}\,dx$$

Definition of Improper Integral II

If f(x) is continuous on [a,b) and discontinuous at x=b,

$$\int_a^b f(x)dx = \lim_{t\to b^-}\int_a^t f(x)dx$$

Similar for continuous on (a,b].

The improper integral is <u>convergent</u> if

The improper integral is <u>divergent</u> if

$$\int_4^8 \frac{1}{\sqrt{8-x}}dx$$

Warning:

$$\int_4^8 \frac{1}{6-x}\,dx$$

$$\int_0^1 \ln(x)\,dx$$

$$\lim_{x\to\infty} \frac{x^7 - 1}{1000000x^6 - 5}$$